2 Mechanical Properties of Solids

Mechanical Properties of Solids

Elastic behaviour, Stress-strain relationship, Hooke's law, Young's modulus, bulk modulus, shear modulus of rigidity, Poisson's ratio; elastic energy.

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MIND MAP : LEARNING MADE SIMPLE

Mechanical Properties of Solids

Elasticity

Property of material by virtue of which it regains it original shape & size after the removal of deforming force

Relation between Y, B, η & σ

(i) $Y = 3B(1 - 2\sigma)$

(ii) $Y = 2\eta(1 + \sigma)$

(iii) $\sigma = \frac{3B - 2\eta}{2\eta + 6B}$

(iv) $\frac{9}{Y} = \frac{1}{B} + \frac{3}{\eta}$ or $Y = \frac{9B\eta}{\eta + 3B}$

Poision's ratio(σ) $= \frac{\text{Laternal strain } (\beta)}{\text{Longitudinal strain } (\alpha)} = \frac{\Delta d / d}{\Delta l / l}$

Value of σ lies between 0 and 0.5

Elastic Potential energy

Elastic potential energy in a stretched wire $= \frac{1}{2} \times$ stress × strain × volume of the wire

Hooke's law

Within the elastic limit, stress is directly proportional to strain. i.e., stress ∝ strain

Stress

Restoring force per unit area i.e., stress $= \frac{F}{A}$

Types

(i) Longitudinal stress

$= \frac{\text{Deforming force applied normal}}{\text{Area}} = \frac{F_n}{A}$

(ii) Volumetric stress

$= \frac{\text{Force Changing Volume}}{\text{Area}} = \frac{F_V}{A}$

(iii) Shearing area or tangential stress

$= \frac{\text{Tangential Force}}{\text{Area}} = \frac{F_t}{A}$

Strain

Ratio of change in configuration to original configuration

Strain $= \frac{\text{change in configuration}}{\text{original configuration}}$

Types

(i) Longitudinal strain $= \frac{\text{change in length}}{\text{original length}} = \frac{\Delta l}{l_o}$

(ii) Volumetric strain $\varepsilon_V = \frac{\text{change in volume}}{\text{original volume}} = \frac{\Delta V}{V_o}$

(iii) Shearing strain = angular displacement of the plane perpendicular to the fixed surface = θ

Types of Modulus of Elasticity

Young's Modulus

Young's modulus of elasticity

$Y = \frac{\text{longitudinal stress}}{\text{longitudinal strain}} = \frac{F / A}{l / \Delta l}$

$Y = \frac{F\Delta l}{Al} = \frac{Mg\Delta l}{\pi r^2 l}$

Bulk Modulus

Bulk modulus or volume modulus of elasticity

$B = \frac{\text{hydraulic stress}}{\text{volume strain}}$

$B = -\frac{\Delta P}{\frac{\Delta V}{V}}; \frac{1}{B}$ = compressibility

Modulus of Rigidity

Rigidity or shear modulus of elasticity

$\eta = \frac{\text{tangential stress}}{\text{shearing strain}}$

$\eta = \frac{\sigma_s}{\varepsilon_s} = \frac{\frac{F}{A}}{\theta} = \frac{F}{A\theta}$

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1. Introduction

A rigid body generally means a hard solid object having a definite shape and size. But in reality, bodies can be stretched, compressed and bent. Even the appreciably rigid steel bar can be deformed when a sufficiently large external force is applied on it. This means that solid bodies are not perfectly rigid. A solid has definite shape and size. In order to change (or deform) the shape or size of a body, a force is required.

2. Deforming Force

A force which produces a change in configuration (size or shape) of the object on applying it, is called a deforming force.

3. Elasticity

Elasticity is that property of the object by virtue of which it regains its original configuration after the removal of the deforming force.

For example, if we stretch a rubber band and release it, it snaps back to its original length.

4. Perfectly Elastic Body

Those bodies which regain its original configuration immediately and completely after the removal of deforming force are called perfectly elastic bodies. The nearest approach to a perfectly elastic body is quartz fibre.

5. Plasticity

If a body does not regains its original size and shape completely and immediately after the removal of deforming force, it is said to be a plastic body and this property is called plasticity.

6. Perfectly plastic body

That body which does not regain its original configuration at all on the removal of deforming force are called perfectly plastic bodies.Putty and paraffin wax are nearly perfectly plastic bodies.

7. Stress

If a body gets deformed under the action of an external force, then at each section of the body an internal force of reaction is set up which tends to restore the body into its original state.

7.1 Definition

The internal restoring force set up per unit area of cross section of the deformed body is called stress.

7.2 Mathematical Form

$$\text{Stress} = \frac{\text{Applied Force}}{Area}$$

Its unit is N/m2 or Pascal.

Its dimensional formula is $[ML^{-1}T^{-2}]$.

7.3 Types of stress

There are three different types of stress

1. Longitudinal Stress

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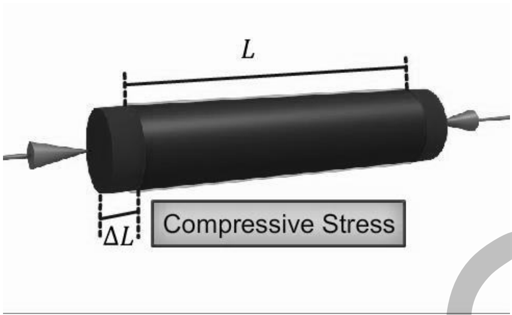
If deforming force is applied normal to the area of cross section, then the stress is called longitudinal stress. It is further categorized in two types

(a) **Tensile stress** If there is an increase in length of the object under the effect of applied force, then stress is called tensile stress.

(b) **Compressional stress** If there is a decrease in length of the object under the effect of applied force, then stress is called compression stress.



2. Tangential or Shearing Stress

If deforming force acts tangentially to the surface of a body, it produces a change in the shape of the body. The tangential force applied per unit area is called tangential stress.

3. Normal Stress

If a body is subjected to a uniform force from all sides, then the corresponding stress is called hydrostatic stress.

8.Strain

When a deforming force acts on a body, the body undergoes a change in its shape and size. The fractional change in configuration is called strain.

8.1 Mathematical Equation

$$\text{Strain} = \frac{\text{change in dimension}}{\text{original dimension}}$$

It has no unit and it is a dimensionless quantity.

According to the change in configuration, the strain is of three types

(1) longitudinal strain$= \frac{\text{change in length}}{\text{original length}}$

(2) Volumetric strain $= \frac{\text{change in volume}}{\text{Original volume}}$

(3) $Shearing\ strain \frac{\text{tangential applied force}}{\text{Area of face}}$

9. Hooke's Law

Robert Hook found that within the elastic limit, the stress is directly proportional to strain. Thus we have

$$\boldsymbol{stress \propto strain}$$

or

$$\boldsymbol{stress = K.strain}$$

where K is the constant of proportionality called "Elastic Modulus" of the material.

There are some materials that do not obey Hooke's law like rubber, human's muscle.

9.1 Types of Modulus of rigidity

9.1.1 Young's Modulus of rigidity (Y)

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It is defined as the ratio of normal stress to the longitudinal strain within the elastic limit.

$$Y = \frac{longitudinal\ stress}{\text{Longitudinal strain}}$$

It has same units as stress because strain does not have any unit. Y is measured in N/m^2 or Pa.

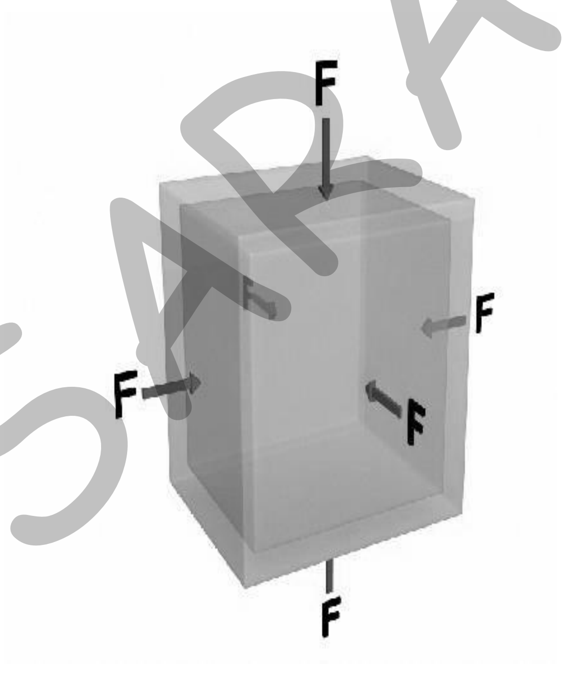
Metals generally have large values of Young's modulus compare to other materials. In scientific terms, the higher the Young's modulus of the material the more elastic it is.

9.1.2 Bulk Modulus of Rigidity

$$\kappa = \frac{normal\ stress}{\text{Volumetric strain}}$$

or $\kappa = \frac{-F/A}{\Delta V/V}$ = - pV/ΔV

The SI unit of bulk modulus is N/m^2



Compressibility

Compressibility of a material is the reciprocal of its bulk modulus of elasticity. Compressibility (C) = 1 / k

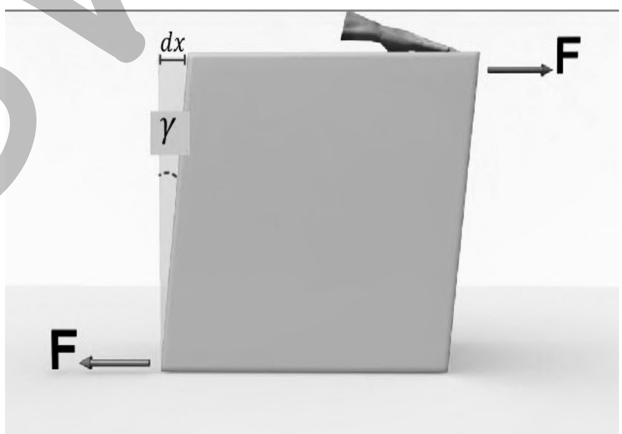
Its SI unit is N-1m 2 and CGS unit is $dyne^{-1}$ cm^2 .

9.1.3 Modulus of rigidity or shear Modulus (η)

$$\eta = \frac{tangential\ stress}{\text{shear strain}}$$

$$\eta = \frac{\frac{F}{A}}{Y} = \frac{F}{AY}$$

$$\eta = \frac{F}{AY}$$



The SI unit of shear modulus is N/m^2

The shear modulus of a material is always considerably smaller than the Young's modulus for it.

10. Limit of elasticity

The maximum value of deforming force for which elasticity is present in the body is called its limit of elasticity.

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11.Stress- strain Curve

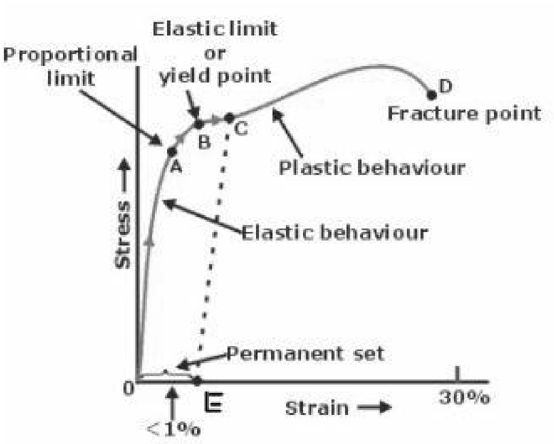

Figure shows the stress-strain curve for a metal wire which is gradually being loaded.

(a) The initial part OA of the graph is a straight line indicating that stress is proportional to strain. Upto the point A, Hooke's law is obeyed. The point A is called the proportional limit. In this region, the wire is perfectly elastic.

(b) After the point A, the stress is not proportional to strain and a curved portion AB is obtained. However, if the load is removed at any point between O and B, the curve is retraced along BAO and the wire attains its original length. The portion OB of the graph is called elastic region and the point B is called elastic limit or yield point. The stress corresponding to B is called yield strength.

(c) Beyond the point B, the strain increases more rapidly than stress. If the load is removed at any point C, the wire does not come back to its original length but traces dashed line. Even on reducing the stress to zero, a residual strain equal to OE is left in the wire. The material is said to have acquired a permanent set. The fact that stress-strain curve is not retraced on reversing the strain is called elastic hysteresis.

(d) If the load is increased beyond the point C, there is large increase in the strain or the length of the wire. In this region, the constrictions (called necks and waists) develop at few points along the length of the wire and the wire breaks ultimately at the point D, called the fracture point.

In the region between B and D, the length of the wire goes on increasing even without any addition of load. This region is called plastic region and material is said to undergo plastic flow or plastic deformation. The stress corresponding to the braking point is called ultimate strength or tensile strength of the material.

12. Elastic after Effect

The bodies return to their original state on the removal of the deforming force. Some bodies return to their original state immediately after the removal of the deforming force while some bodies take longer time to do so. The delay in regaining the original state by a body on the removal of the deforming force is called elastic after effect.

13. Elastic Fatigue

The property of an elastic body by virtue of which its behavior becomes less elastic

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under the action of repeated alternating deforming force is called elastic fatigue.

14. Ductile Materials

The materials which have large plastic range of extension are called ductile materials. Such materials undergo an irreversible increase in length before snapping. So they can be drawn into thin wires. For e.g. copper, silver, iron, aluminium etc.

15 Brittle Materials

The materials which have very small range of plastic extension are called brittle materials. Such materials break as soon as the stress is increased beyond the elastic limit. For e.g. cast iron, glass, ceramics etc.

16. Elastomers

The materials for which strain produced is much larger than the stress applied, with in the limit of elasticity are called elastomers, e.g., rubber, the elastic tissue of aorta, the large vessel carrying blood from heart. etc. Elastomers have no plastic range.

17. Elastic Potential Energy of stretched wire

When a wire is stretched, interatomic forces come into play which opposes the change. Work has to done against these restoring forces. The work done in stretching the wire is stored in it as its elastic potential energy.

18. Poisson's Ratio

When a deforming force is applied at the free end of a suspended wire of length 1 and diameter D, then its length increases by Δl but its diameter decreases by ΔD. Now two types of strains are produced by a single force.

(i) Longitudinal strain = $\frac{\Delta l}{l}$

(ii) Lateral strain = $\frac{-\Delta D}{D}$

∴ Poisson's Ratio (σ) = $\frac{Lateral\ strain}{longitudinal\ strain}$ =

$$= \frac{\frac{-\Delta D}{D}}{\frac{\Delta l}{l}} = -\frac{l\Delta D}{D\Delta l}$$

The negative sign shows that longitudinal and lateral strains are in opposite sense.

As Poisson's ratio is the ratio of two strains, it has no units and dimensions.

The theoretical value of Poisson's ratio lies between – 1 and 0.5. Its practical value lies between 0 and 0.5

19. Applications of elasticity

The elastic behavior of materials plays an important role in everyday life. All engineering designs require precise knowledge of the elastic behavior of materials. For example while designing a building, the structural design of the columns, beams and supports require knowledge of strength of materials used.

A bridge has to be designed such that it can withstand the load of the flowing traffic, the force of winds and its own weight. Similarly, in the design of buildings use of beams and

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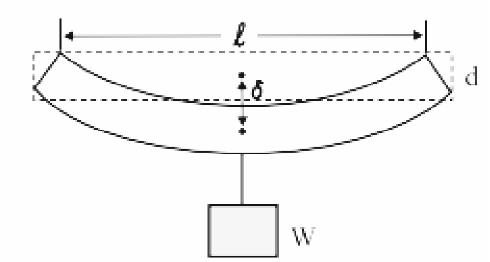
columns is very common. In both the cases, the overcoming of the problem of bending of beam under a load is of prime importance. The beam should not bend too much or break. Let us consider the case of a beam loaded at the centre and supported near its ends as shown in Fig.



A bar of length l, breadth b, and depth d when loaded at the centre by a load W sags by an amount given by $\delta = W l^3/(4bd^3Y)$

Bending can be reduced by using a material with a large Young's modulus Y. Depression can be decreased more effectively by increasing the depth d rather than the breadth b. But a deep bar has a tendency to bend under the weight of a moving traffic, hence a better choice is to have a bar of I-shaped cross section. This section provides a large load bearing surface and enough depth to prevent bending. Also this shape reduces the weight of the beam without sacrificing its strength and hence reduces the cost.

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QUESTION ALIKE

Very Short Answer Type Questions :

1. Why are girders given I shape?

Sol. To reduce stress at top and bottom of girders.

2. The length of wire increases by 1 mm under 1 N. What will be increase in length under 2 N?

Sol. 2 mm

3. What is the SI unit of volumetric strain?

Sol. No unit

4. Which of the three *Y*, *B* or *G* is possible in all the three state of matter (solid, liquid and gas)?

Sol. Bulk modulus of elasticity (*B*) only.

5. Which type of strain is there, when a spiral spring is stretched by a force?

Sol. Shear strain

6. State Hooke's law.

Sol. Within elastic limit,

stress $\propto$ strain

$\Rightarrow$ stress = $E \times$ strain

7. What is the dimensional formula of Young's modulus?

Sol. $[ML^{-1}T^{-2}]$

8. Define bulk modulus of elasticity.

Sol. The ratio of volume stress to volume strain is called bulk modulus.

9. What do you mean by Poisson's ratio?

Sol. The ratio of the lateral strain to the longitudinal strain is called the poisson's ratio.

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10. What is the value of modulus of rigidity for a liquid?

Sol. Zero

Short Answer Type Questions :

11. Why is a spring made of steel, not of copper?

Sol. A spring will be better one, if a large restoring force is set up in it on being deformed, which in turn depends upon the elasticity of material of the spring. Since the Young's modulus of elasticity of steel is more than that of copper, hence steel is used.

12. An elastic wire is cut to half its original length. How would it affect the maximum load that the wire can support?

Sol. Breaking load = breaking stress × area, is free from the length of elastic wire.

13. Which is more elastic-rubber or steel? Explain.

Sol. Steel, because it sustains more deforming force.

14. Calculate the force required to increase length by 1% of a rod of area of cross-section 10^{-3} m^2. Modulus of elasticity is 1.2×10^{12} Nm^{-2}.

Sol. $F = YA \cdot \frac{\Delta l}{l}$

$F = 1.2 \times 10^4$ N

15. Explain the terms (i) Young's modulus (ii) Bulk modulus.

Sol. (i) Young's modulus : The ratio of the longitudinl stress to the longitudinal strain is called Young's modulus.

(ii) Bulk modulus : The ratio of volume stress to volume strain is called bulk modulus.

16. A thick rope of density ρ and length L is hung from a rigid support. The Young's modulus of the material of rope is Y. What is the increase in length of the rope due to its own weight?

Sol. Let A be the area of cross-section of the rope. Weight of the rope, $F = AL\rho g$. It will be acting at the centre of gravity of the rope which lies at a distance $\frac{L}{2}$ from rigid support. Therefore,

$$Y = \frac{\text{normal stress}}{\text{longitudinal strain}}$$

$$= \frac{AL\rho g/A}{\frac{\Delta L}{L/2}} \text{ or } \Delta L = \frac{L^2 \rho g}{2Y}$$

17. Define elastic limit.

Sol. No body is perfectly elastic. However, a body behaves as a perfectly elastic body and recovers its original configuration completely when the deforming force does not exceed a particular limit. This limit is called the elastic limit.

18. When the pressure on a sphere is increased by 80 atm then its volume decreases by 0.01%. Find the Bulk modulus of elasticity of the material of sphere.

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Sol. $P = 80 \times 1.013 \times 10^5$ N/m^2

$$\frac{\Delta V}{V} = \frac{0.01}{100}$$

$$\therefore \quad B = \frac{PV}{\Delta V} = \frac{80 \times 1.013 \times 10^5}{\frac{0.01}{100}} = 8.1 \times 10^{10} \text{ N/m}^2$$

19. A specimen of oil having an initial volume of 600 cm^3 is subjected to a pressure increase of 3.6×10^6 Pa and the volume is found to decrease by 0.45 cm^3. What is the Bulk modulus of the material?

Sol. $B = \frac{PV}{\Delta V}$

$$= \frac{3.6 \times 10^6 \times 600}{0.45}$$

$$= 4.8 \times 10^9 \text{ N/m}^2$$

20. Compare the mechanical properties of a steel cable, made by twisting many thin wires together, with the properties of a solid steel rod of the same diameter. What advantages does each have?

Sol. Refer theory

21. The material in human bones and elephant bones is essentially same, but an elephant has much thicker legs. Explain why, in terms of breaking stress.

Sol. $\text{Stress} = \frac{\text{force}}{\text{area}}$

22. If a metal wire has its length doubled and its diameter tripled, then by what factor does its Young's modulus change?

Sol. Young's modulus of material is independent of stress and strain.

23. Calculate the longest length of steel wire that can hang vertically without breaking. Breaking stress for steel $= 7.982 \times 10^8$ N/m^2 and density for steel $d = 8.1 \times 10^3$ kg/m^3.

Sol. $\text{Breaking stress} = \frac{\text{force}}{\text{area}} = \frac{mg}{a}$

$$\Rightarrow \quad \text{Breaking stress} = \frac{\rho l a g}{a}$$

$$l = \frac{\text{Breaking stress}}{\rho g}$$

$$l = \frac{7.982 \times 10^8}{8.1 \times 10^3 \times 9.8} \text{ m}$$

$$l = 1 \times 10^4 \text{ m}$$

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24. A copper wire of length 2.2 m and a steel wire of length 1.6 m, both of diameter 3.0 mm, are connected end to end. When stretched by a load, the net elongation is found to be 0.70 mm. Obtain the load applied.

Sol. The copper and steel wires are under a tensile stress because they have the same tension (equal to the load *w*) and the same area of cross-section *A*. We have, stress = strain × Young's modulus. Therefore,

$$\frac{w}{A} = Y_c \cdot \frac{\Delta L_c}{L_c} = Y_s \cdot \frac{\Delta L_s}{L_s}$$

Where the subscripts *c* and *s* refer to copper and stainless steel respectively, or,

$$\frac{\Delta L_c}{\Delta L_s} = \left(\frac{Y_s}{Y_c}\right) \times \left(\frac{L_c}{L_s}\right)$$

Given, L_c = 2.2 m, L_s = 1.6 m

From table...... $Y_c = 1.1 \times 10^{11}\ \text{Nm}^{-2}$ and $Y_s = 2.0 \times 10^{11}\ \text{Nm}^{-2}$

$$\frac{\Delta L_c}{\Delta L_s} = \left(\frac{2.0 \times 10^{11}}{1.1 \times 10^{11}}\right) \times \left(\frac{2.2}{1.6}\right) = 2.5$$

The total elongation is given to be

$\Delta L_c + \Delta L_s = 7.0 \times 10^{-4}$ m

Solving the above equations,

$\Delta L_c = 5.0 \times 10^{-4}$ m and $\Delta L_s = 2.0 \times 10^{-4}$ m

Therefore, $w = \dfrac{(A \times Y_c \times \Delta L_c)}{L_c}$

$= \pi(1.5 \times 10^{-3})^2 \times [(5.0 \times 10^{-4} \times 1.1 \times 10^{11})/2.2]$

$= 1.8 \times 10^2$ N

25. A 2 m long wire is stretched by 0.5 cm. Calculate the elastic potential energy per unit volume if the Young's modulus of the material of the wire is $Y = 8 \times 10^{10}\ \text{N/m}^2$.

Sol. $u = \dfrac{1}{2}(\text{stress}) \times \text{strain}$

$$= \frac{1}{2}\text{ Young's modulus} \times (\text{strain})^2$$

$$= \frac{1}{2} \times 8 \times 10^{10} \times \left(\frac{0.5 \times 10^{-2}}{20}\right)^2$$

$$= 4 \times 10^{10} \times \frac{25 \times 10^{-4}}{400}$$

$= 2.5 \times 10^4\ \text{J/m}^3$

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Long Answer Type Questions :

26. A metal wire 3.50 m long and 0.70 mm in diameter has given the following test. A load weighing 20 N was originally hung from the wire to keep it straight. The position of the lower end of the wire was read on a scale as load was added.

Added Load (N)	Scale Reading (cm)
0	3.02
10	3.07
20	3.12
30	3.17
40	3.22
50	3.27
60	3.32
70	4.27

(i) Graph these values, plotting the increases in length horizontally and the added load vertically.

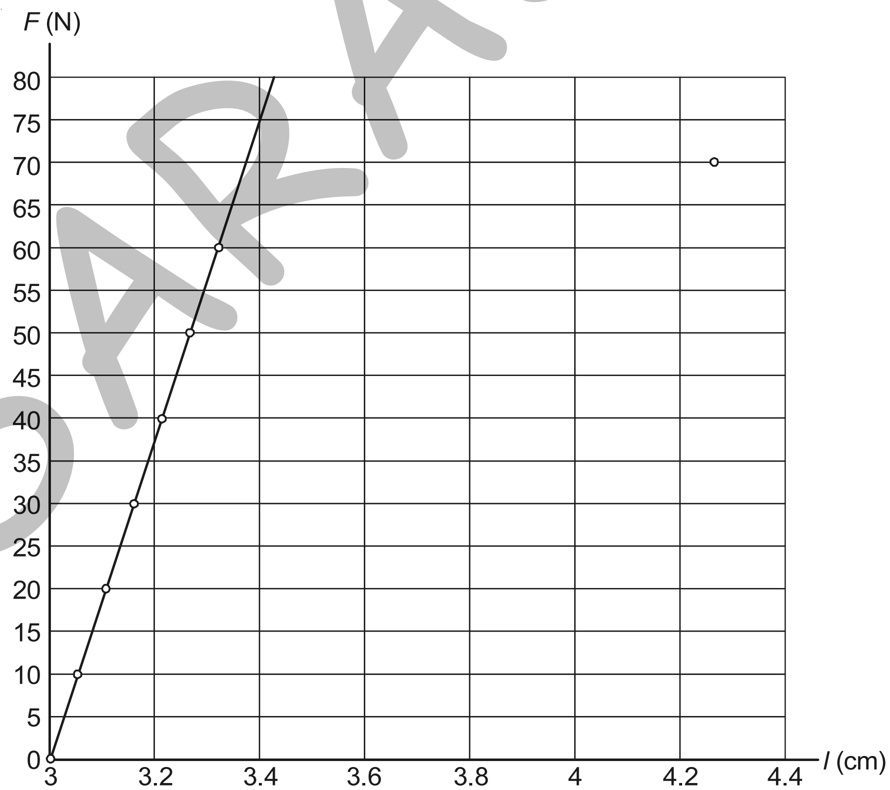
(ii) Calculate the value of Young's modulus.

(iii) The proportional limit occurred at a scale reading of 3.34 cm. What was the stress at this point?

Sol. $F_{\perp} = \left(\frac{YA}{l_0}\right)\Delta l$ so the slope of the graph in part (a) depends on Young's modulus.

$F_{\perp}$ is the total load, 20 N plus the added load.

(i) The graph is given in figure



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(ii) The slope is $\dfrac{60\text{ N}}{(3.32-3.02)\times10^{-2}\text{ m}} = 2.0\times10^4\text{ N/m}$

$$Y = \left(\frac{l_0}{\pi r^2}\right)(2.0\times10^4\text{ N/m}) = \left(\frac{3.50\text{ m}}{\pi[0.35\times10^{-3}\text{ m}]^2}\right)(2.0\times10^4\text{ N/m}) = 1.8\times10^{11}\text{ Pa}$$

(iii) The stress is $F_\perp/A$. The total load at the proportional limit is 60 N + 20 N = 80 N

$$\text{stress} = \frac{80\text{ N}}{\pi(0.35\times10^{-3}\text{ m})^2} = 2.1\times10^8\text{ Pa}$$

The value of Y we calculated is close to the value for iron, nickel and steel.

27. A brass rod with a length of 1.40 m and a cross-sectional area of 2.00 cm^2 is fastened end to end to a nickel rod with length L and cross-sectional area 1.00 cm^2. The compound rod is subjected to equal and opposite pulls of magnitude 4.00 × 10^4 N at its ends.

(i) Find the length L of the nickel rod if the elongation of the two rods are equal.

(ii) What is the stress in each rod?

(iii) What is the strain in each rod?

Sol. Each piece of the composite rod is subjected to a tensile force of 4.00 × 10^4 N.

(i) $Y = \dfrac{F_\perp l_0}{A\Delta l}$ so $\Delta l = \dfrac{F_\perp l_0}{YA}$

$\Delta l_b = \Delta l_n$ gives that $\dfrac{F_\perp l_{0,b}}{Y_b A_b} = \dfrac{F_\perp l_{0,n}}{Y_n A_n}$ (b for brass and n for nickel); $l_{0,n} = L$

But the $F_\perp$ is same for both, so

$$l_{0,n} = \frac{Y_n}{Y_b}\frac{A_n}{A_b} l_{0,b}$$

$$L = \left(\frac{21\times10^{10}\text{ Pa}}{9.0\times10^{10}\text{ Pa}}\right)\left(\frac{1.00\text{ cm}^2}{2.00\text{ cm}^2}\right)(1.40\text{ m}) = 1.63\text{ m}$$

(ii) Stress $= \dfrac{F_\perp}{A} = \dfrac{T}{A}$

Brass : stress = T/A = (4.00 × 10^4 N)/(2.00 × 10^{-4} m^2) = 2.00 × 10^8 Pa

Nickel : stress = T/A = (4.00 × 10^4 N)/(1.00 × 10^{-4} m^2) = 4.00 × 10^8 Pa

(iii) Y = stress/strain and strain = stress/Y

Brass : strain = (2.00 × 10^8 Pa)/(9.0 × 10^{10} Pa) = 2.22 × 10^{-3}

Nickel : strain = (4.00 × 10^8 Pa)/(21 × 10^{10} Pa) = 1.90 × 10^{-3}

Larger Y means less Δl and smaller A means greater Δl, so the two effects largely cancel and the lengths don't differ greatly. Equal Δl and nearly equal l means the strains are nearly the same. But equal tensions and A differing by a factor of 2 means the stresses differ by a factor of 2.

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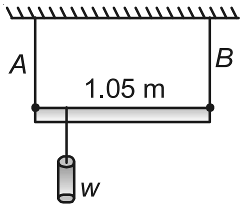
28. A 1.05 m long rod of negligible weight is supported at its ends by wires *A* and *B* of equal lengths as shown in figure. The cross-sectional area of *A* is 2.00 mm^2 and that of *B* is 4.00 mm^2. Young's modulus for wire *A* is 1.80 × 10^{11} Pa; and that for *B* is 1.20 × 10^{11} Pa. At what point along the rod should a weight *w* be suspended to produce



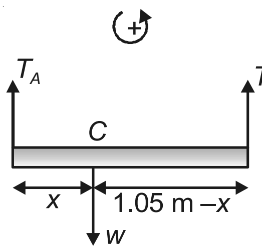
(i) Equal stress in *A* and *B* and

(ii) Equal strains in *A* and *B*?

Sol. (i) Stress = $F_\perp/A$, so equal stress implies T/A same for each wire.

$T_A/2.00\ \text{mm}^2 = T_B/4.00\ \text{mm}^2$ so $T_B = 2.00T_A$

The question is where along the rod to hang the weight in order to produce this relation between the tensions in the two wires. Let the weight be suspended at point *C*, a distance *x* to the right of wire *A*. The free-body diagram for the rod is given in the figure.



$$\sum \vec{\tau}_C = 0$$
$$+T_B(1.05\ \text{m} - x) - T_A x = 0$$

But $T_B = 2.00T_A$ so $2.00T_A(1.05\ \text{m} - x) - T_A x = 0$

$2.10\ \text{m} - 2.00x = x$ and $x = 2.10\ \text{m}/3.00 = 0.70$ m (measured from *A*).

(ii) Y = stress/strain gives that strain = stress/Y = $F_\perp/AY$.

Equal strain thus implies

$$\frac{T_A}{(2.00\ \text{mm}^2)(1.80\times10^{11}\ \text{Pa})} = \frac{T_B}{(4.00\ \text{mm}^2)(1.20\times10^{11}\ \text{Pa})}$$

$$T_B = \left(\frac{4.00}{2.00}\right)\left(\frac{1.20}{1.80}\right)T_A = 1.333T_A$$

The $\sum \vec{\tau}_C = 0$ equation still gives $T_B(1.05\ \text{m} - x) - T_A x = 0$

But now $T_B = 1.333T_A$ so $(1.333T_A)(1.05\ \text{m} - x) - T_A x = 0$

$1.40\ \text{m} = 2.33x$ and $x = 1.40\ \text{m}/2.33 = 0.60$ m (measured from *A*)

Wire *B* has twice the cross-sectional area so it takes twice the tension to produce the same stress. For equal stress the moment arm for T_B (0.35 m) is half that for T_A (0.70 m), since the torques must be equal. The smaller *Y* for *B* partially compensates for the larger area in determining the strain and for equal strain the moment arms are closer to being equal.

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Multiple Choice Questions

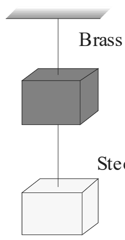
1. The breaking stress of wire depends upon
(a) length of the wire
(b) radius of the wire
(c) material of the wire
(d) shape of the cross section

2. A metal ring of initial radius r and cross-sectional area A is fitted onto a wooden disc of radius $R > r$. If young's modulus of the metal is Y then the tension in the ring is :
(a) $\frac{AYR^2}{r}$ (b) $\frac{AY(R-r)}{r}$
(c) $\frac{Y}{A}\frac{(R-r)}{r}$ (d) $\frac{Yr}{AR}$

3. The length of a metal wire is L_1 when the tension in it is T_1 and is L_2, when the tension is T_2. The natural length of the wire is :
(a) $\frac{L_1+L_2}{2}$ (b) $\sqrt{(L_1L_2)}$
(c) $\frac{L_1T_2-L_2T_1}{T_2-T_1}$ (d) $\frac{L_1T_1+L_2T_2}{T_2+T_1}$

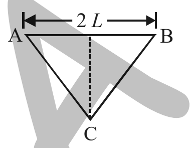
4. A body of mass M is attached to the lower end of a metal wire, whose upper end is fixed. The elongation of the wire is 1
(i) Loss in gravitational potential energy of M is Mgl
(ii) The elastic potential energy stored in the wire is Mgl
(iii) The elastic potential energy stored in the wire is 1/2 Mgl
(iv) Heat produced is 1/2 Mgl
(a) (i), (iii), (iv) (b) (ii), (iii), (iv)
(c) (i) (ii), (iii) (d) all

5. If the ratio of lengths, radii and Young's modulus of steel and brass wires shown in the figure are a, b and c respectively. the ratio between the increase in lengths of brass and steel wires would be



(a) $\frac{b^2a}{2c}$ (b) $\frac{bc}{2a^2}$
(c) $\frac{3b^2c}{2a}$ (d) $\frac{a}{2b^2c}$

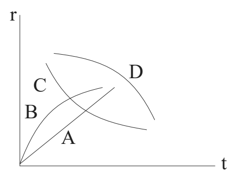
6. A wire of length 2L and radius r is stretched between A and B without the application of any tension. If Y is the Young's modulus of the wire and it is stretched like ACB, then the tension in the wire will be :



(a) $\frac{\pi r^2 Yd^3}{2L^2}$ (b) $\frac{\pi r^2 Yd^2}{2L^2}$
(c) $\frac{\pi r^2 Y 2L^2}{d^2}$ (d) $\frac{\pi r^2 Y 2L}{d}$

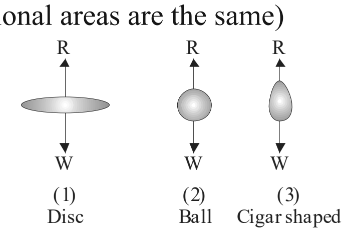
7. Which of the following relations is true ?
(a) $3Y = K(1-\mu)$ (b) $\mu = \frac{Y}{2\eta} - 1$
(c) $K = \frac{9Y\eta}{Y+\eta}$ (d) $\mu = (6K+\eta)Y$

8. A spherical ball is dropped in a long column of a viscous liquid. The speed of the ball as a function of time may be best represented by the graph



(a) A (b) B
(c) C (d) D

9. When a body falls in air, the resistance of air depends to a great extent on the shape of the body, 3 different shapes are given Identify the combination of air resistance which truly represent the physical situation. (The cross-sectional areas are the same)



(a) 1<2<3 (b) 2<3<1

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(c) 3<2<1 (d) 3<1<2

10. The rate of steady volume flow of water through a capillary tube of length 'l' and radius 'r' under a pressure difference of P is V. This tube is connected with another tube of the same length but half the radius is series. Then the rate of steady volume flow through them is (the pressure difference across the combination is P)

(a) $\frac{V}{16}$ (b) $\frac{V}{17}$

(c) $\frac{16V}{17}$ (d) $\frac{17V}{16}$

11. A liquid is contained in a vertical tube of semicircular cross-section. The contact angle is zero. the force of surface tension on the curved part and on the flat part are in ratio :

(a) $1:1$ (b) $1:2$

(c) $\pi:2$ (d) $2:\pi$

12. A thin metal disc of radius r floats on water surface and bends the surface downwards along the perimeter making an angle θ with vertical edge of the disc. If the disc displaces a weight of water W and surface tension of water is T, then the weight of metal disc is :

(a) $2\pi T + W$ (b) $2\pi T\cos\theta - W$

(c) $2\pi T\cos\theta + W$ (d) $W - 2\pi T\cos\theta$

13. When water droplets merge to form a bigger drop

(a) energy is liberated

(b) energy is absorbed

(c) energy is neither liberated nor absorbed

(d) energy may either be liberated or absorbed depending on the nature of the liquid.

14. A long cylindrical glass vessel has a smaller hole of radius 'r' at its bottom. The depth to which the vessel can be lowered vertically in the deep water bath (surface tension T) without any water entering inside is

(a) $4T/\rho rg$ (b) $3T/\rho rg$

(c) $2T/\rho rg$ (d) $T/\rho rg$

15. The specific heat of a substance varies according to the equation, $C = 3t^2 cal/g°C$. The amount of heat required to raise the temperature of 10 g of substance from $5\,°C$ to $10°C$ is :

(a) 8750 cal (b) 4375 cal

(c) 9550 cal (d) 8125 cal

16. Calorie is defined as the amount of heat required to raise temperature of 1g of water by $1°C$ and it is defined under which of the following conditions ?

(a) From $14.5°C$ to $15.5°C$ at 760 mm of Hg

(b) From $98.5°C$ to $99.5°C$ at 760 mm of Hg

(c) From $13.5°C$ to $14.5°C$ at 76 mm of Hg

(d) From $3.5°C$ to $4.5°C$ at 76 mm of Hg

17. 500g of water and 100 g of ice at $0°C$ are in a calorimeter whose water equivalent is 40 g, 10 g of steam at $100°C$ is added to it. Then water in the calorimeter is : (Latent heat of $ice = 80\,cal/g$. Latent heat of steam $= 540\,cal/g$

(a) 580 g (b) 590 g

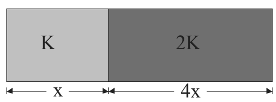
(c) 600 g (d) 610 g

18. A cylinder of radius R made of a material of thermal conductivity K_1 is surrounded by a cylindrical shell of inner radius R and outer radius 2R made of material of thermal conductivity K_2. The two ends of the combined systems are maintained at two different temperatures. There is no loss of heat across the cylindrical surface and the system is in steady state. The effective thermal conductivity of the system is :

(a) $K_1 + K_2$ (b) $\frac{K_1 + 3K_2}{4}$

(c) $\frac{K_1K_2}{K_1 + K_2}$ (d) $\frac{3K_1 + K_2}{4}$

19. The temperature of the two outer surfaces of a composite slab consisting of two materials having coefficient of thermal conductivities K and 2K, thickness x and 4x respectively are T_2 and $T_1\left(T_2 > T_1\right)$. The rate of heat transfer through the slab, in a steady state is $f\left[\frac{A\left(T_2 - T_1\right)K}{x}\right]$, the value of f is



(a) 1 (b) $\frac{1}{2}$

(c) $\frac{2}{3}$ (d) $\frac{1}{3}$

20. Which of the following is incorrect regarding the first law of thermodynamics :

(a) It is a restatement of the principle of conservation of energy.

(b) It is not applicable to any cyclic process

(c) It introduces the concept of the

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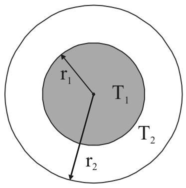
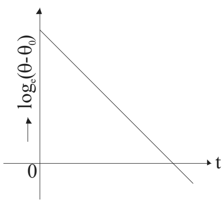
entropy
(d) It introduces the concept of the internal energy.

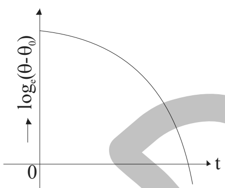
21. The figure shows a system of two concentric spheres of radii r_1 and r_2 and kept at temperatures T_1 and T_2, respectively. The radial rate of flow of heat in a substance between the two concentric spheres is proportional to

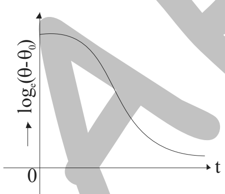


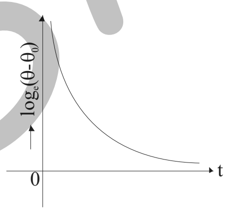
(a) $\frac{r_1 r_2}{(r_2 - r_1)}$ (b) $(r_2 - r_1)$
(c) $(r_2 - r_1)(r_1 r_2)$ (d) $In\left(\frac{r_2}{r_1}\right)$

22. A liquid in a beaker has temperature $\theta(t)$ at time t and θ_0 is temperature of surroundings, then according to Newton's law of cooling the correct graph between $\log_e(\theta - \theta_0)$ and t is :

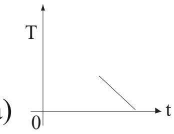
(a)


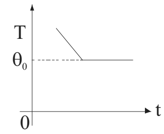
(b)


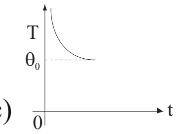
(c)


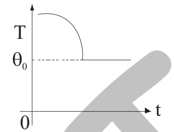
(d)


23. If a piece of metal is heated to temperature θ and then allowed to cool in a room which is at temperature θ_0, the graph between the temperature T of the metal and time t will be closest to

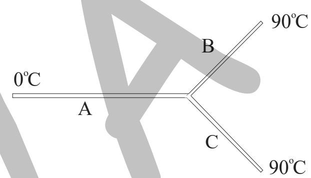
(a)


(b)


(c)


(d)


24. Three rods made of the same material and having the same cross section have been joined as shown in the figure. Each rod is of the same length. The left and right ends are kept at $0°C$ and $90°C$ respectively. The temperature of the junction of the three rods will be :



(a) $45°C$ (b) $60°C$
(c) $30°C$ (d) $20°C$

25. A spherical black body with a readius of 12 cm radiates 450 W power at 500 K. If the radius wer halved and the temperature doubled, the power radiated in watt would be
(a) 225 (b) 450
(c) 900 (d) 1800

26. The intensity of radiation emitted by the sun has its maximum value at a wavelength of 510 nm and the emitted by the north star has the maximum value at 350 nm. If these stars behave like black bodies, then the ratio of the surface temperature of the sun and north star is :
(a) 1.46 (b) 0.69
(c) 1.21 (d) 0.83

27. Two bodies A and B have emissivity of 0.01 and 0.81 respectively. The outer surface area of the two bodies are the same. The two bodies emit total radiation power at the same rate. The temperature of body A is 1000 K, then temperature of body B is :
(a) 100 K (b) 1000/3K
(c) 300K (d) $\frac{2000}{3}K$

28. A body cools in 7 minute from $60°C$ to $40°C$. What will be its temperature after the next 7 minute ? The temperature of the surroundings is $10°C$. Assume that Newton's law of cooling holds good throughout the process.
(a) $18°C$ (b) $24°C$

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(c) $28°C$ (d) $32°C$

29. A copper sphere is suspended in an evacuated chamber maintained at 300 K. The sphere is maintained at a constant temperature of 500 K by heating it electrically. A total of 210 W of electric power is needed to do it. When the surface of the copper sphere is completely blackened, 700 W is needed to maintain the same temperature of the sphere. The emissivity of copper :
(a) 0.8 (b) 0.9
(c) 0.6 (d) 0.3

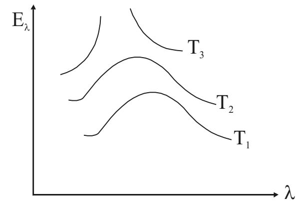
30. The spectral energy distribution of the sum has a maximum at $4753\overset{o}{A}$. If the temperature of the sun is 6050 K, the temperature of a star for which this maximum is at $9506\overset{o}{A}$ is :
(a) 3025 k (b) 4050 k
(c) 6050 k (d) 7045 k

31. Assuming the sun to be a spherical body of radius R at a temperature of TK, evaluate the total radiant power, incident on Earth, at a distance r from the sun
(a) $\pi r_0^2 \sigma T^4 / r^2$ (b) $r_0^2 R^2 \sigma T^4 / 4\pi r^2$
(c) $R^2 \sigma T^4 / r^2$ (d) $4\pi r_0^2 \sigma T^4 / r^2$

32. An ideal black body at room temperature is thrown into a furnace. It is observed that
(a) Initially it is the darkest body and at later times the brightest.
(b) It is the darkest body at all times
(c) It cannot be distinguished at all times
(d) Initially it is the darkest body and at later times it cannot be distinguished

33. Three discs A, B and C having radii 2m, 4m and 6m respectively are coated with carbon black on their outer surfaces. The wavelength corresponding to maximum intensity are 300 nm, 400 nm and 500 nm respectively. The power radiated by them are Q_A, Q_B and Q_C respectively
(a) Q_A is maximum (b) B_B is maximum
(c) Q_C is maximum (d) $Q_A = Q_B = Q_C$

34. Variation of radiant energy emitted by sun, filament of tungsten lamp and welding arc is a function of its wavelength is shown in figure. Which of the following option is the correct match ?



(a) Sun-T_3, tungsten filament-T_1, welding arc-T_2
(b) Sun-T_2, tungsten filament-T_1, welding arc-T_2
(c) Sun-T_3, tungsten filament-T_2, welding arc T_1
(d) Sun-T_1, tungsten filament-T_2, welding arc T_3

35. In which of the following process, convection does not take place primarily
(a) sea and land breeze
(b) boiling of water
(c) heating air around a furnace
(d) warming of glass of bulb due to filament

36. A spherical body of area A and emissivity $e = 0.6$ is kept inside a perfectly black body. Total heat radiated by the body at temperature T
(a) $0.4\,AT^4$ (b) $0.8\,AT^4$
(c) $0.6\,AT^4$ (d) $1.0\,AT^4$

37. The heat radiated per unit area in 1 hour by a furnace whose temperature is 3000 K is $\left(\sigma = 5.7\times10^{-8} Wm^{-2}K^{-4}\right)$
(a) $1.7\times10^{10} J$ (b) $1.1\times10^{12} J$
(c) $2.8\times10^{8} J$ (d) $4.6\times10^{6} J$

38. A hot body, obeying Newton's law of cooling of cooling down from its peak value $80°C$ t an ambient temperature of $30°C$. It takes 5 minutes in cooling down from $80°C$ to $40°C$. How much time will it take to cool down from $62°C$ to $32°C$?
(Given in 2=0.693, In 5=1.609)
(a) 3.75 minutes (b) 8.6 minutes
(c) 9.6 minutes (d) 6.5 minutes

39. Hot water cools from $60°C$ to $50°C$ in the first 10 minutes and to $42°C$ in the next 10 minutes. The temperature of the surrounding is :
(a) $25°C$ (b) $10°C$
(c) $15°C$ (d) $20°C$

40. An aluminium sphere of 20 cm diameter is heated from $0°C$ to $100°C$. Its volume changes by (given that coefficient of linear expansion for aluminium $\alpha_{A\ell} = 23\times10^{-6} / °C$)
(a) 2.89 cc (b) 9.28 cc
(c) 49.8 cc (d) 28.9 cc

41. Two rods one of aluminium and the other of steel, having initial length l_1 and l_2 are

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connected together to form a single rod of length $(l_1 + l_2)$. The coefficients of linear expansion for aluminium and steel are α_{Al} and α_S respectively. If the length of each rod increases by the same amount when their temperature are raised by $t°C$, then the ratio $\frac{\ell_1}{\ell_1 + \ell_2}$ is :

(a) $\frac{\gamma_{Al}}{\gamma_S + \gamma_S}$ (b) $\frac{\gamma_S}{\gamma_{Al} + \gamma_{Al}}$

(c) $\frac{\gamma_S}{\gamma_{Al} + \gamma_S}$ (d) $\frac{\gamma_S}{\gamma_S + \gamma_{Al}}$

42. A cylindrical vessel is filled with water up to height H. A hole is bored in the wall at a depth h from the free surface of water. For maximum range, h is equal to

(a) $\frac{H}{4}$ (b) $\frac{H}{2}$

(c) $\frac{3H}{4}$ (d) H

43. At two points on a horizontal tube of varying cross section carrying water, the radii are 1 cm. The pressure difference between these points is 4.9 cm of water, How much liquid flows through the tube per seconds?

(a) 100 c.c per sec (b) 80 c.c. per sec

(c) 50 c.c. per sec (d) 70 c.c. per sec

44. Bernoulli's equation for steady, non-viscous, incompressible flow express the

(a) conservation of angular momentum

(b) conservation of density

(c) conservation of momentum

(d) conservation of energy

45. A cylindrical tank is filled with water to a level of 3m. A hole is opened at a height of 52.5 cm bottom. The ratio of the area of the hole to that of cross-sectional area of the cylinder is 0.1. The square of the speed with which water is coming out from the orifice is

(Take g= 10 m/s^2)

(a) 50 m^2/s^2 (b) 40 m^2/s^2

(c) 51.5 m^2/s^2 (d) 50.5 m^2/s^2

46. A tank full of water has a small hole at its bottom. If on-fourth of the tank is emptied in t_1 seconds and the remaining three fourths of the tank is emptied in t_2 seconds, then ratio $\left(\frac{t_1}{t_2}\right)$ is

(a) $\sqrt{3}$ (b) $\sqrt{2}$

(c) $\frac{2-\sqrt{2}}{\sqrt{2}}$ (d) $\frac{2-\sqrt{3}}{3}$

47. In old age arteries carrying blood in the human body become narrow resulting in an increase in the blood pressure. This follows from

(a) Pascal's law (b) Stroke' law

(c) Bernoulli's principle

(d) Archimedes' principle

48. A large open tank has two holes in its wall, One is a square hole of side a at a depth of x from the top and the other is a circular hole of radius r at depth 4x from the top. When the tank is completely filled with water, the quantities of water flowing out per second from both holes are the same. Then r is equal to

(a) $2\pi a$ (b) $\frac{a}{2\pi}$

(c) $\frac{a}{\pi}$ (d) $\frac{a}{\sqrt{2\pi}}$

49. Bernoulli's principle is not involved in the working/explanation of

(a) movement of spinning ball

(b) carburetor of automobile

(c) blades of a kitchen mixer

(d) dynamic lift of an aeroplane

50. The working of venturimeter is based on

(a) Torricelli's law (b) Bernoulli's theorem

(c) Archimedes' principle

(d) Stroke' law

Answer Key

1. C	2. B	3. C
4. A	5. C	6. B
7. B	8. B	9. C
10. B	11. C	12. C
13. A	14. C	15. A
16. A	17. C	18. B
19. D	20. C	21. A
22. A	23. C	24. B
25. D	26. B	27. B
28. C	29. D	30. A
31. D	32. A	33. B
34. A	35. D	36. D
37. A	38. B	39. B
40. D	41. C	42. B
43. C	44. D	45. A
46. D	47. C	48. D
49. C	50. B	

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