CHAPTER-01

NUMBER SYSTEMS

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NUMBER SYSTEM

'THE ULTIMATE FAMILY TREE'

REAL NUMBERS - Dadaji (Grandfather)

RATIONAL NUMBER (Good child)

→ any number that can be expressed in $\frac{p}{q}$ form where $q \neq 0$, p & q are co-prime

(further has family of 3)

IRRATIONAL NUMBER (Bad child)

→ Any number that cannot be expressed in $\frac{p}{q}$ form.

→ **NATURAL NUMBERS**:-

No.s that begin with 1, 2, 3, ------- etc.

→ **WHOLE NUMBER**

No.s that begin with 0, 1, 2, 3, ---- etc.

→ **INTEGERS**

- set of numbers, consist of Natural No.s, negative of Natural No.s

→ +ve INTEGERS   -ve INTEGERS

IMPORTANT SYMBOLS/ ABBREVIATIONS

N:- The set of Natural No.s

W:- set of whole No.s

Z:- set of integers

Q:- set of Rational No.

R:- set of Real No.

$N \rightarrow W \rightarrow Z \rightarrow Q \rightarrow R$

$N \subset W \subset Z \subset Q \subset R$

SOME IMPORTANT TERMS AND IMPORTANT POINTS

- The set N of Natural No.s is infinite, i.e it has unlimited members.
- N has smallest element namely '1'.
- N has No largest element i.e, give me any natural number, we can find the bigger no. from the given no.
- '0' is Not the member of Set N.
- Set of whole Numbers is infinite
- smallest whole No. is '0'.
- set of whole No.s has No largest member
- every natural no. is a whole No.
- Non-zero smallest whole No. is 1.
- The set Z of integers has neither the greatest nor the least element.
- every natural number is an integer
- every whole Number is an integer

- CONSECUTIVE NUMBER:- A series of natural no. each differing by one is called consecutive ~~integer~~ number, eg. 50, 51, 52 etc.

- PRIME NO.:- No. that has only two factors namely one and itself eg:- 2, 3, 5, 7, 11, 13 etc.
  - ↳ 1 is not a prime no.
  - ↳ only even prime no. is 2, all are odd.
  - ↳ smallest prime no. is 2

• **TWIN-PRIME** :- Pair of prime no.s is said to be twin prime if they differ by 2 for eg:- (3,5), (11,13), (17,19), (29,31), (41,43), (71,73) are all twin-prime.

• **COMPOSITE NUMBERS** :- Those No.s which can be expressed as product of primes, 4, 6, 12 etc.

↳ 1 is neither prime, Nor composite

↳ 1 is not composite.

↳ 4 is the smallest Composite No.

• **CO-PRIME NO.** :- A pair of No. is said to be co-prime if the numbers have no common factor other than one.

for eg:- 29 and 31 are co-prime.

• **PERFECT NUMBER** :- A no. is said to be perfect if it is equal to sum of its factors other than itself. for eg:-

$$6 = (1+2+3)$$

$$28 = (1+2+4+7+14)$$

∴ 6 and 8 are perfect Numbers.

CLASSIFICATION OF RATIONAL & IRRATIONAL

TERMINATING DECIMALS

↳ rational no.s with a finite decimal part after finite no.s of steps are known as terminating decimals

eg: $\frac{1}{2} = 0.5$, $\frac{7}{8} = 0.875$ etc

NON TERMINATING DECIMALS

– Those no.s in which the division process never comes to an end.

Repeating or recurring

block of digits repeat itself

eg:- $0.333 \ldots \left(\frac{1}{3}\right)$

$\frac{2}{3} = 0.6666 \ldots$

$0.1717 \ldots$

$0.186186 \ldots$

Pure recurring decimals

→ A decimal in which all the digits after the decimal point are repeated.

eg: $0.66 \ldots$, $0.\overline{16}$, $0.\overline{123}$

Mixed recurring decimals

↳ A decimal in which at least one of the digits after decimal repeats eg:- $0.1\overline{6}$, $0.35\overline{2}$ etc

Non-repeating

$\frac{22}{7} = 3.14159265$

$\sqrt{2} = 1.414 \ldots$

$\sqrt{3} = 1.732 \ldots$

- every Integer is a Rational Number
- every terminating decimal is a rational No.
- every recurring decimal is a rational No.
- A non terminating repeating decimal is called recurring decimal.
- Between any two rational No.s there are infinite no. of rational no.s. &
- every rational No. can be represented in the form of terminating or non-terminating recurring decimal.
- A no. is irrational no., it has a non-terminating & non-repeating decimal representation.

- REAL NUMBERS :- rational no.s and irrational No.s taken together form the set of real numbers. denoted by R, $\sqrt{3}$, 2, -5, etc.

NOTE :- π is defined as ratio of circumference of a circle to the length of the diameter. π is an irrational no. since value of π is $\Rightarrow \pi = 3.14159265$ -- which is neither terminating non repeating

IMPORTANT PROPERTIES OF IRRATIONAL NUMBERS

Property-1

Negative of an irrational No. is an irrational No.

Property 2

Sum of rational and irrational no. is irrational No.

Property 3

Product of non-zero rational and irrational No. is irrational.

EXCEPTION :- 0 is rational No., $\sqrt{3}$ is irrational

$\therefore$ $0 \times \sqrt{3} = 0$ which is rational

Property 4

Division of non-zero rational No. and an irrational No. is irrational

EXCEPTION :- 0 is rational No., $\sqrt{3}$ is irrational No.

$\therefore \frac{0}{\sqrt{3}} = 0$ which is rational no.

Property-5

Sum of two irrational no. may be rational or irrational.

eg $\sqrt{2}+\sqrt{3}$ is irrational, but $(2+\sqrt{3})+(2-\sqrt{3})$

$\Rightarrow$ 4 is rational

Property-6

- Difference of two irrational no. may be rational or irrational

eg:- $\sqrt{2}+2,\ 2-\sqrt{2} \Rightarrow 2\sqrt{2}$ is irrational

but $2+\sqrt{2},\ -2+\sqrt{2} = 4$ is rational

Property-7

- Product of two irrational No. may be rational or irrational

eg:- $\sqrt{2}\times\sqrt{3}=\sqrt{6}$ is an irrational

But $(\sqrt{3}-2)(\sqrt{3}+2) = (\sqrt{3})^2-(2)^2 = 3-4=-1$ is rational No.

Property-8

- Division of two irrational no. may be rational or irrational.

eg $\sqrt{18}\div\sqrt{3} = \sqrt{6}$ is irrational, $\sqrt{125}\div\sqrt{5} = 5$ is rational

IMPORTANT POINTS

- A rational no. is either terminating or non-terminating but repeating & can be put in p/q form.

SOME USEFUL LAWS

LAW 1 if a & b are positive real numbers, then

$\sqrt{ab} = \sqrt{a} \times \sqrt{b}$,

eg:- $\sqrt{5 \times 3} = \sqrt{5} \times \sqrt{3}$ etc.

LAW 2 if a & b are +ve real No.s, then $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

eg:- $\sqrt{\frac{7}{5}} = \frac{\sqrt{7}}{\sqrt{5}}$ etc.

LAW 3 if a & b are positive real no.s, then

$(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b}) = a-b$

$\Rightarrow (\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3}) = (\sqrt{5})^2 - (\sqrt{3})^2 = 5-3 = 2$

LAW 4 if a & b are positive real no.s, then

$(a+\sqrt{b})(a-\sqrt{b}) = a^2 - b$

eg:- $(6+\sqrt{2})(6-\sqrt{2}) = (6)^2 - (\sqrt{2})^2 = 36-2 = 34$

LAW 5 if a & b are +ve real no.s,

$(\sqrt{a}+\sqrt{b})(\sqrt{c}+\sqrt{d}) = \sqrt{ac} + \sqrt{ad} + \sqrt{bc} + \sqrt{bd}$

eg:- $(\sqrt{5}+\sqrt{3})(\sqrt{6}+\sqrt{7}) = \sqrt{5 \times 6} + \sqrt{5 \times 7} + \sqrt{3 \times 6} + \sqrt{3 \times 7}$

LAW 6 if a & b are positive real Nos, then

$(\sqrt{a}+\sqrt{b})^2 = a + 2\sqrt{ab} + b$.

LAWS OF EXPONENTS

LAW-1

$a^m \times a^n = a^{m+n}$

LAW-2

$\frac{a^m}{a^n} = a^{m-n}$

LAW-3

$(a^m)^n = a^{mn} = (a^n)^m$

LAW-4

$(ab)^n = a^n b^n$

LAW-5

$\left(\frac{a}{b}\right)^m = \frac{a^n}{b^n}, b \neq 0$

LAW-6

$a^0 = 1$

LAW-7

$a^m \times b^m = (ab)^m$

LAW-8

$a^{-m} = \frac{1}{a^m}$

LAW-9

$\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$

LAW-10

$\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$

<u>NOTE</u> :- $(\sqrt{a}) = (a)^{\frac{1}{2}}$. (i.e square root means power half)

$\sqrt[3]{a} = (a)^{\frac{1}{3}}$ (i.e cube root means power $\frac{1}{3}$)

$\therefore$ in general $\sqrt[n]{a} = a^{\frac{1}{n}}$

$\boxed{\text{Also } \sqrt[n]{a^m} = a^{\frac{m}{n}}}$

Trick to be used be a lawyer.

Lawyer

Truth → facts

False → Proof

TYPE-1 STATE TRUE OF FALSE WITH REASONS

Q1.) is -2 a rational No.?

Q2.) is every whole number a rational number?

Q3.) is every integer a whole number?

Q4.) is zero an integer?

Q5.) every rational number is a whole no.?

TYPE-II :- FINDING RATIONAL NUMBERS B/W TWO RATIONAL NO.S, & IRRATIONAL NO. B/W TWO RATIONAL NO.

Q6.) Find three rational number between the following.

a.) $\frac{1}{2}$ and $\frac{1}{3}$

b.) -2 and 5

c.) 7 and 8

d.) 10 and 11

e.) 3 and 8

f.) $\frac{1}{2}$ and $\frac{1}{7}$

g.) 4 and 5

h.) $\frac{1}{2}$ and $\frac{1}{6}$

i.) 3 and 4

j.) $\frac{-3}{11}$ and $\frac{8}{11}$

k.) $\frac{-3}{13}$ and $\frac{9}{13}$

l.) $-\frac{2}{3}$ and $\frac{1}{4}$

m.) -2 and 6

n.) 3.2 and 3

n.) $\frac{1}{5}$ and $\frac{1}{7}$

Q7.) Find two rational No.s between the following irrational numbers.

a.) 2.1010--- and 2.50500500---

b.) 1.303003000--- and 1.404004000---

c.) 0.8910100100--- and 0.9101001000---

d.) 6.101001000--- and 5.202002000---

e.) 0.232332333 2 3333--- and 0.21 2112111 ---

f.) 0.515115111 5--- and 0.535335 335---

g.) 0.303003003--- and 0.301001 0001---

Q8.) Find two irrational No. b/w the following Rational Numbers

i.) $\frac{1}{7}$ and $\frac{2}{7}$

ii.) $\frac{1}{3}$ and $\frac{1}{6}$

iii.) 0.12 and 0.13

iv.) $\frac{5}{7}$ and $\frac{9}{11}$

v.) 2 and 2.5

vi.) $\sqrt{2}$ and $\sqrt{3}$

vii.) 0.5 and 0.55

viii.) 0.1 and 0.12

TYPE-III BASED ON TERMINATING and NON-TERMINATING NUMBERS

Q9) Express the No.s in $\frac{p}{q}$ form

i.) 0.15 ii.) 0.675 iii.) 0.00026

iv.) 15.75 v.) 8.0025 vi.) -25.876

(Ans i.) $\frac{3}{20}$ ii.) $\frac{27}{40}$ iii.) $\frac{13}{50000}$ iv.) $\frac{63}{4}$ v.) $\frac{3201}{400}$

vi.) $-\frac{411}{16}$)

Q10) Express each of the following in $\frac{p}{q}$ form

i.) $0.\overline{1}$ ii.) $0.\overline{2}$ iii.) $0.\overline{4}$ iv.) $0.\overline{5}$

(Ans → i.) $\frac{1}{9}$ ii.) $\frac{2}{9}$ iii.) $\frac{4}{9}$ iv.) $\frac{5}{9}$)

TRICK TO CHECK YOUR ANSWER IN ABOVE TYPE :- $\frac{\text{No. of digits repeated in Numerator}}{10-1}$ (if one digit is repeated.)

for eg :- $0.\overline{1} \Rightarrow \frac{1}{10-1} = \frac{1}{9}$

Q11) Express each of following in $\frac{p}{q}$ form.

i.) $0.\overline{35}$ ii.) $0.\overline{585}$ iii.) $0.\overline{621}$

iv.) $0.\overline{37}$ v.) $0.\overline{75}$

SAME TRICK AS ABOVE ONLY write 100 ~~or~~ or 1000 in denominator acc. to No. of digits repeated

Ans i.) $\frac{35}{99}$ ii.) $\frac{585}{999}$ iii.) $\frac{621}{999}$ iv.) $\frac{37}{99}$ v.) $\frac{75}{99}$

Q12) express each of the following in $\frac{p}{q}$ form.

(i) $5.\overline{2}$ (ii) $23.\overline{43}$ (iii) $0.3\overline{2}$ (iv) $0.1\overline{23}$

(v) $0.003\overline{52}$ (vi) $4.3\overline{2}$ (vii) $15.7\overline{12}$

(viii) $125.\overline{3}$

Ans) (i) $\frac{47}{9}$ (ii) $\frac{2320}{99}$ (iii) $\frac{29}{90}$ (viii) $\frac{376}{3}$
(iv) $\frac{111}{900}$ (v) $\frac{349}{99000}$ (vi) $\frac{389}{90}$ (vii) $\frac{5185}{330}$

TYPE-IV IDENTIFY AS RATIONAL OR IRRATIONAL

Q13) Determine if the following are irrational or rational.

(i) $\sqrt{7}$ (ii) $\sqrt{4}$ (iii) $2+\sqrt{3}$ (iv) $\sqrt{3}+\sqrt{5}$

(v) $(\sqrt{2}-2)^2$ (vi) $(2-\sqrt{2})(2+\sqrt{2})$ (vii) $\sqrt{5}-2$

(viii) $\sqrt{23}$ (ix) $\sqrt{225}$ (x) 0.3796

(xi) $3\sqrt{18}$ (xii) $\sqrt{1.44}$ (xiii) $\sqrt{\frac{9}{27}}$

(xiv) $-\sqrt{64}$ (xv) $\sqrt{100}$ (xvi) $\sqrt{45}$

Q14) Find which variables x, y, z etc. represent rational or irrational No.s

i.) $x^2 = 5$ (IRR)

ii.) $y^2 = 9$ (R.)

iii.) $z^2 = 0.04$ (R)

iv.) $u^2 = \frac{17}{4}$ (Irr)

v.) $v^2 = 3$ (IRR)

vi.) $w^2 = 27$ (Irr)

vii.) $t^2 = 0.4$ (IRR)

Q15.) Give example of two rational No.s whose

(Ans)

i.) difference is a rational number $(\sqrt{2}, \sqrt{2})$

ii.) difference is an irrational number $(4\sqrt{3}, 2\sqrt{3})$

iii.) Sum is a rational Number $(\sqrt{5}, -\sqrt{5})$

iv.) sum is an irrational Number $(2\sqrt{5}, 3\sqrt{5})$

v.) product is a rational number $(\sqrt{8}, \sqrt{2})$

vi.) product is an irrational no. $(\sqrt{2}, \sqrt{3})$

vii) quotient is a rational no. $(\sqrt{8}, \sqrt{2})$

viii.) quotient is an irrational no. $(\sqrt{2}, \sqrt{3})$

TYPE-5 QUESTIONS BASED ON EXPONENTS

Q16.) evaluate each of the following — Ans

a.) $\left(\frac{2}{11}\right)^4 \times \left(\frac{11}{3}\right)^2 \times \left(\frac{3}{2}\right)^3$ — $\left(\frac{6}{121}\right)$

b.) $\left(\frac{1}{2}\right)^5 \times \left(\frac{-2}{3}\right)^4 \times \left(\frac{3}{5}\right)^{-1}$ — $\left(\frac{5}{486}\right)$

c.) $2^{55} \times 2^{60} - 2^{97} \times 2^{18}$ — (0)

d.) $\left(\frac{2}{3}\right)^2 \times \left(\frac{2}{5}\right)^{-3} \times \left(\frac{3}{5}\right)^2$ — $\left(\frac{5}{2}\right)$

Q17.) Simplify — Ans

a.) $(3a^4b^3)(18a^3b^5)$ — $(54a^7b^8)$

b.) $\frac{3a^7b^6}{18a^6b^8}$ — $\left(\frac{1}{6}ab^{-2}\right)$

c.) $\left(\frac{-2a^2}{b^3}\right)^3$ — $\left(\frac{-8a^6}{b^9}\right)$

d.) $\frac{(4 \times 10^7)(6 \times 10^{-5})}{8 \times 10^4}$ — $\left(\frac{3}{100}\right)$

e.) $\frac{4ab^2(-5ab^3)}{10a^2b^2}$ — $(-2b^3)$

Q18.) Simplify each of the following

i.) $\frac{7^{n+2} - 3 \times 7^{n+1}}{20 \times 7^n - 2 \times 7^n}$

Since $a^n \times a^m = a^{m+n}$

$$\Rightarrow \frac{7^n \times 7^2 - 3\times 7\times 7^n}{20\times 7^n - 2\times 7^n}$$ (taking 7^n common from Nr & Dr)

$$\Rightarrow \frac{7^n(49-21)}{7^n(20-2)} \Rightarrow \frac{28}{18} \Rightarrow \boxed{\frac{14}{9}}$$ Ans

ii.) $\dfrac{5^{n+3} - 6\times 5^{n+1}}{9\times 5^n - 2^2\times 5^n}$ $\Rightarrow$ (Hint: $\dfrac{5^n\times 5^3 - 6\times 5^n\times 5}{9\times 5^n - 2^2\times 5^n} = 19$)

iii.) $\dfrac{16\times 2^{n+1} - 4\times 2^n}{16\times 2^{n+2} - 2\times 2^{n+2}}$ (same as above) (Ans $\rightarrow \frac{1}{2}$)

iv.) $\dfrac{3^n\times 9^{n+1}}{3^{n-1}\times 9^{n-1}}$

v.) $\dfrac{5\times 25^{n+1} - 25\times 5^{2n}}{5\times 5^{2n+3} - (25)^{n+1}}$

vi.) $\dfrac{5^{n+3} - 6\times 5^{n+1}}{9\times 5^n - 2^2\times 5^n}$

vii.) $\dfrac{6\times(8)^{n+1} + 16\times(2)^{3n-2}}{10\times(2)^{3n+1} - 7\times(8)^n}$

Q19) EXAM if $\dfrac{9^n\times 3^2\times 3^n - 27^n}{3^{3m}\times 2^3} = \dfrac{1}{27}$, prove that $m-n=1$.

we have

$$\Rightarrow \frac{9^n\times 3^2\times 3^n - (3^3)^n}{3^{3m}\times 2^3} = \frac{1}{3^3}$$

$$\Rightarrow \frac{(3^2)^n\times 3^2\times 3^n - 3^{3n}}{3^{3m}\times 2^3} = \frac{1}{3^3}$$

$$\Rightarrow \frac{3^{2n} \times 3^2 \times 3^n - 3^{3n}}{3^{3m} \times 2^3} = \frac{1}{3^3}$$

$$\Rightarrow \frac{3^{2n} \times 3^2 \times 3^n - 3^{3n}}{3^{3m} \times 8} = \frac{1}{3^3}$$

$$\Rightarrow \frac{3^{2n+2+n} - 3^{3n}}{3^{3m} \times 8} = \frac{1}{3^3}$$

$$\Rightarrow \frac{3^{3n+2} - 3^{3n}}{3^{3m} \times 8} = \frac{1}{3^3} \Rightarrow \frac{3^{3n}(3^2-1)}{3^{3m}(8)} = \frac{1}{3^3}$$

$$\Rightarrow \frac{3^{3n}(9-1)}{3^{3m}(8)} = \frac{1}{3^3} \Rightarrow \frac{3^{3n}}{3^{3m}} = \frac{1}{3^3}$$

$$\Rightarrow 3^{3n-3m} = 3^{-3}$$

$\Rightarrow$ on comparing

$$3n - 3m = -3$$

$$\Rightarrow 3(n-\cancel{3}m) = -\cancel{3}$$

$$\Rightarrow n - m = -1 \Rightarrow \boxed{m - n = 1}$$

Hence proved

TYPE-VI PROVING QUESTIONS BASED ON EXPONENTS

Q20) Assuming that x is a positive real no.s a, b and c are rational no.s, show that.

i.) $\left(\dfrac{x^b}{x^c}\right)^a \times \left(\dfrac{x^c}{x^a}\right)^b \times \left(\dfrac{x^a}{x^b}\right)^c = 1$ (hint:- use rules of exponents & expand.)

ii.) $\left(\dfrac{x^a}{x^b}\right)^{\frac{1}{ab}} \times \left(\dfrac{x^b}{x^c}\right)^{\frac{1}{bc}} \times \left(\dfrac{x^c}{x^a}\right)^{\frac{1}{ac}} = 1$ (seperate using GOLDEN-RULE ALAG-ALAG)

iii.) $\left(\dfrac{x^a}{x^b}\right)^{a^2+b^2+ab} \left(\dfrac{x^b}{x^c}\right)^{b^2+bc+c^2} \left(\dfrac{x^c}{x^a}\right)^{c^2+ca+a^2} = 1$ (use Ch-2 identity)

iv.) $\left(\dfrac{x^a}{x^b}\right)^{a+b} \left(\dfrac{x^b}{x^c}\right)^{b+c} \left(\dfrac{x^c}{x^a}\right)^{c+a} = 1$ (use identity)

v.) $\left(\dfrac{x^a}{x^b}\right)^{a+b-c} \left(\dfrac{x^b}{x^c}\right)^{b+c-a} \left(\dfrac{x^c}{x^a}\right)^{c+a-b} = 1$

vi.) $\left(\dfrac{x^a}{x^{-b}}\right)^{a^2+b^2-ab} \left(\dfrac{x^b}{x^{-c}}\right)^{b^2+c^2-bc} \left(\dfrac{x^c}{x^{-a}}\right)^{c^2+a^2-ca} = x^{2(a^3+b^3+c^3)}$

vii.) $\dfrac{x^{a(b-c)}}{x^{b(a-c)}} \div \left(\dfrac{x^b}{x^a}\right)^c = 1$

viii) $\dfrac{(x^{a+b})^2 (x^{b+c})^2 (x^{c+a})^2}{(x^a x^b x^c)^4} = 1$ $\rightarrow$ (hint:- $(x^{a+b+c})^4 = x^{4a+4b+4c}$)

Q21) Show that

i) $$\frac{1}{1+x^{b-a}+x^{c-a}}+\frac{1}{1+x^{a-b}+x^{c-b}}+\frac{1}{1+x^{b-c}+x^{a-c}}=1$$

Ans) Multiplying the Nr & Dr of three terms on LHS by x^a, x^b, x^c respectively, we obtain.

$$\text{LHS}=\frac{1}{1+x^{b-a}+x^{c-a}}+\frac{1}{1+x^{a-b}+x^{c-b}}+\frac{1}{1+x^{b-c}+x^{a-c}}$$

$$\Rightarrow \frac{x^a}{x^a+x^{b-a+a}+x^{c-a+a}}+\frac{x^b}{x^b+x^{a-b+b}+x^{c-b+b}}+\frac{x^c}{x^c+x^{b-c+c}+x^{a-c+c}}$$

$$\Rightarrow \frac{x^a}{x^a+x^b+x^c}+\frac{x^b}{x^a+x^b+x^c}+\frac{x^c}{x^a+x^b+x^c}$$

$$\Rightarrow \frac{x^a+x^b+x^c}{x^a+x^b+x^c}=\boxed{1}\ \text{Ans}$$

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ii) $$\frac{1}{1+x^{a-b}}+\frac{1}{1+x^{b-a}}=1$$ (Hint:- Same as above)

iii) $$\frac{1}{1+x^{b-a}+x^{c-a}}+\frac{1}{1+x^{a-b}+x^{c-b}}+\frac{1}{1+x^{b-c}+x^{a-c}}=1$$

Q22) Prove that

i.) $\frac{a^{-1}}{a^{-1}+b^{-1}} + \frac{a^{-1}}{a^{-1}-b^{-1}} = \frac{2b^2}{b^2-a^2}$

(Hint :- use laws of exponents.)

ii.) $\frac{a+b+c}{a^{-1}b^{-1}+b^{-1}c^{-1}+c^{-1}a^{-1}} = abc$

iii.) $(a^{-1}+b^{-1})^{-1} = \frac{ab}{a+b}$

iv.) If $abc = 1$.

then $\frac{1}{1+a+b^{-1}} + \frac{1}{1+b+c^{-1}} + \frac{1}{1+c+a^{-1}} = 1$

Soln $\Rightarrow$ $\frac{1}{1+a+\frac{1}{b}} + \frac{1}{1+b+\frac{1}{c}} + \frac{1}{1+c+\frac{1}{a}}$

(Now since $abc = 1 \therefore c = \frac{1}{ab}$) (also $\frac{1}{c} = ab$)

$\Rightarrow \frac{1}{\frac{b+ab+1}{b}} + \frac{1}{1+b+ab} + \frac{1}{1+\frac{1}{ab}+\frac{1}{a}}$

$\Rightarrow \frac{b}{b+ab+1} + \frac{1}{b+ab+1} + \frac{ab}{b+ab+1}$

$\Rightarrow \frac{b+ab+1}{b+ab+1} = \boxed{1}$ Hence proved v.) if $abc = 1$, show that

$$\frac{1}{1+ab^{-1}} + \frac{1}{1+b+c^{-1}} + \frac{1}{1+c+a^{-1}} = 1$$

TYPE-VII FINDING THE VALUE OF X

Q23) find the value of x if

i.) $5^{x-3} \times 3^{2x-8} = 225$

soln) $5^{x-3} \times 3^{2x-8} = 5^2 \times 3^2$ (By prime factorisation)

$\Rightarrow$ on comparing

$x-3=2$, $2x-8=2$

$\hookrightarrow \boxed{x=5}$ $\hookrightarrow \boxed{x=5}$ $\therefore \boxed{x=5}$

---

ii.) $2^{x-5} = 256$ (Ans→13) iii) $2^{x+3} = 4^{x-1}$ ($x=5$)

iv.) $7^{2x+3} = 1$ (Ans→ $-\frac{3}{2}$) v.) $2^{5x+3} = 8^{x+3}$ ($x=3$)

vi.) $2^{3x-7} = 256$ (Ans→5) vii.) $4^{2x} = \frac{1}{32}$ ($x = -\frac{5}{4}$)

viii) $4^{x-1} \times (0.5)^{3-2x} = \left(\frac{1}{8}\right)^x$ (Ans→ $x = \frac{5}{7}$)

ix.) $2^{3x-7} = 256$ (Ans→ $x=5$)

Q24) solve the equations to find values of x (HOTS)

i.) $2^{2x+1} = 17 \cdot 2^x - 2^3$

$\Rightarrow 2^{2x} \cdot 2 = 17 \cdot 2^x - 2^3$

$\Rightarrow 2 \cdot (2^x)^2 = 17 \cdot 2^x - 8$

$\Rightarrow 2 \cdot (2^x)^2 - 17 \cdot 2^x + 8 = 0$

Put $2^x = y$

$\Rightarrow 2y^2 - 17y + 8 = 0$

$\Rightarrow 2y^2 - 16y - y + 8 = 0$

$\Rightarrow 2y(y-8) - (y-8) = 0$

$\Rightarrow (y-8)(2y-1) = 0 \Rightarrow \boxed{y = 8}$ or $\boxed{y = \frac{1}{2}}$

But $y = 8 \Rightarrow 2^x = 8 \Rightarrow 2^x = 2^3$

$\therefore \boxed{x = 3}$

or $y = \frac{1}{2} \Rightarrow 2^x = \frac{1}{2} \Rightarrow 2^x = 2^{-1}$

$\therefore \boxed{x = -1}$

---

ii) $5^{2x+1} = 6 \cdot 5^x - 1$ (Hint take $5^x = y$) (Ans → $-1, 0$)

iii) $3^{2x+4} + 1 = 2 \cdot 3^{x+2}$ ($3^x = y$) (Ans → -2)

iv.) $2^{2x} - 2^{x+3} + 2^4 = 0$ (Ans → 2)

v.) $9^{x+2} = 720 + 9^x$ (Ans → $x = 1$)

TYPE-VIII FINDING VALUES ON PRIME FACTORISATION

Q25) if a, b, c are distinct positive prime integers such that $a^2b^3c^4 = 49392$, find values of a, b, c

2	49392
2	24696
2	12348
2	6174
2	3087
3	1029
7	343
7	49
7	7
	1

Now :-

$49392 = 2^4 \times 3^2 \times 7^3$

$\therefore a^2b^3c^4 = 49392$

$\Rightarrow a^2b^3c^4 = 2^4 \times 3^2 \times 7^3$

$\Rightarrow a^2b^3c^4 = 3^2 \times 7^3 \times 2^4$

$\Rightarrow \boxed{a = 3}, \boxed{b = 7}$ and $\boxed{c = 4}$

Q26) if $1176 = 2^a \times 3^b \times 7^c$ find a, b, c

(Ans → 3, 1, 2)

Q27) Given $4725 = 3^a 5^b 7^c$, find

i.) integral values of a, b and c

ii.) value of $2^{-a} 3^b 7^c$

(Ans → i.) $a = 3, b = 2, c = 1$ ii.) $\frac{63}{8}$)

Q28) if $a = xy^{p-1}$, $b = xy^{q-1}$ and $c = xy^{r-1}$ prove that $a^{q-r} b^{r-p} c^{p-q} = 1$

(Hint put a, b, c in LHS)

Q29) if $a = x^{m+n} y^{l}$; $b = x^{n+l} y^{m}$, $c = x^{l+m} y^{m}$

prove that $a^{m-n} \, b^{n-l} \, c^{l-m} = 1$

Q30) if $x = a^{m+n}$, $y = a^{n+l}$ & $z = a^{l+m}$,

prove that $x^{m} y^{n} z^{l} = x^{n} y^{l} z^{m}$

(Hint:- solve LHS & RHS separately)

TYPE-IX LAWS OF EXPONENTS

Q31) simplify each of the following

i.) $(625)^{-1/4}$ (Ans→ $\frac{1}{5}$)

ii.) $\left(\frac{256}{81}\right)^{5/4}$ (Ans→ $\frac{1024}{243}$)

iii.) $\sqrt[5]{(32)^{-3}}$ (Ans→ $\frac{1}{8}$)

iv.) $\left[\left\{(625)^{-\frac{1}{2}}\right\}^{-\frac{1}{4}}\right]^{2}$ (Ans→ 5)

v.) $(256)^{-\left(4^{-\frac{3}{2}}\right)}$ (Ans→ $\frac{1}{2}$.)

vi) $\frac{4}{(216)^{-\frac{2}{3}}} + \frac{1}{(256)^{-\frac{3}{4}}} + \frac{1}{(243)^{-\frac{1}{5}}}$

vii) $\sqrt{\frac{1}{4}} + (0.01)^{-\frac{1}{2}} - (27)^{\frac{2}{3}}$ (Ans → $\frac{3}{2}$)

viii) $\left(\frac{1}{4}\right)^{-2} - 3(8)^{\frac{2}{3}} + \left(\frac{9}{16}\right)^{-\frac{1}{2}}$ (Ans → $\frac{16}{3}$)

ix) $\dfrac{(25)^{\frac{3}{2}} \times (243)^{3/5}}{(16)^{\frac{5}{4}} \times (8)^{4/3}}$ (Ans → $\frac{3375}{512}$)

x) $\left(\frac{81}{16}\right)^{-\frac{3}{4}} \times \left[\left(\frac{25}{9}\right)^{-3/2} \div \left(\frac{5}{2}\right)^{-3}\right]$ (Ans → 1)

xi) $(\sqrt{4})^{-3/4}$ (Ans → $\frac{1}{2^{3/4}}$)

xii) $(\sqrt{5})^{-3} \times (\sqrt{2})^{-3}$ ($\frac{10^{1/2}}{100}$)

xiii) $(25)^{-1/3} \times \sqrt[3]{16}$ (Ans $\frac{2}{5} \times 10^{1/3}$)

xiv) $(\sqrt{4})^{-7} \times (\sqrt{2})^{-5}$ (Ans → $\frac{2^{\frac{1}{2}}}{2^{10}}$)

Q32) Simplify

i.) $\sqrt{x^{-2}y^{3}}$ (Ans $\frac{y^{3/2}}{x}$)

ii.) $\left(x^{-2/3}y^{-1/2}\right)^{2}$ (Ans → $\frac{1}{x^{4/3}y}$)

iii.) $\sqrt[4]{\sqrt[3]{x^{2}}}$ (Ans → $x^{\frac{1}{6}}$)

iv.) $\sqrt[3]{xy^{2}} \div x^{2}y$ ($x^{-\frac{5}{3}}y^{-\frac{1}{3}}$)

v.) $(\sqrt{x})^{-\frac{2}{3}}\sqrt{y^{4}} \div \sqrt{xy^{-1/2}}$ (Ans → $\frac{y^{9/4}}{x^{5/6}}$)

Q33) If x, y, z are positive real Numbers show that $\sqrt{x^{-1}y} \times \sqrt{y^{-1}z} \times \sqrt{z^{-1}x} = 1$

Q34) If $\left(\frac{x^{-1}y^{2}}{x^{3}y^{-2}}\right)^{1/3} \div \left(\frac{x^{6}y^{-3}}{x^{-2}y^{3}}\right)^{1/2} = x^{a}y^{b}$ prove that $a+b=-1$, where x and y are diff positive primes.

(Hint : find value of a & b i.e $a = -\frac{16}{3}$, $b = \frac{13}{3}$ ~~them~~ then add & show)

Q35) Find value of x

i.) if $25^{x-1} = 5^{2x-1} - 100$

(Hint $5^{2x-2} - 5^{2x-1} = -100$
$\Rightarrow 5^{2x-1}(5^{-1}-1) = -100$) (Ans $\to x = 2$)

---

ii.) $\sqrt{\left(\frac{3}{5}\right)^{1-2x}} = 4\frac{17}{27}$ (Ans $\to \frac{7}{2}$)

---

iii.) $\sqrt[3]{\left(5^{0}+\frac{2}{3}\right)^{2}} = (0.6)^{3-2x}$ (Ans $\to x = \frac{11}{6}$)

---

iv.) $2^{3}(5^{0}+3^{2x}) = 8\frac{8}{27}$ (Ans $\to x = -\frac{3}{2}$)

---

v.) $3(2^{x}+1) - 2^{x+2} + 5 = 0$ (Ans $\to x = 3$)

vi) $2^{5x} \div 2^{x} = \sqrt[5]{2^{20}}$ (Ans → 1)

vii) $(2^3)^4 = (2^2)^x$ (Ans → 6.)

viii) $5^{x-2} \times 3^{2x-3} = 135$ (Ans → 3.)

ix) $2^{x-7} \times 5^{x-4} = 1250$ (Ans → 8)

x) $\left(\sqrt[3]{4}\right)^{2x+\frac{1}{2}} = \frac{1}{32}$ (Ans → -4)

xi) $\left(\sqrt{\frac{3}{5}}\right)^{x+1} = \frac{125}{27}$ (Ans → -7)

Q36) Solve the equations

i) $3^{x+1} = 27 \times 3^4$ (Ans → 6)

ii) $4^{x-1} \times (0.5)^{3-2x} = \left(\frac{1}{8}\right)^x$ (Ans → $\frac{5}{7}$)

iii) $\sqrt{\frac{a}{b}} = \left(\frac{b}{a}\right)^{1-2x}$ (Ans → $\frac{3}{4}$)

iv) $27^x = \frac{9}{3^x}$ (Ans → $\frac{1}{2}$)

Q37) Prove that

i) $\left[\left\{\frac{x^{a(a-b)}}{x^{a(a+b)}}\right\} \div \left\{\frac{x^{b(b-a)}}{x^{b(b+a)}}\right\}\right]^{a+b} = 1$

ii) $\left(x^{\frac{1}{a-b}}\right)^{\frac{1}{a-c}} \left(x^{\frac{1}{b-c}}\right)^{\frac{1}{b-a}} \left(x^{\frac{1}{c-a}}\right)^{\frac{1}{c-b}} = 1$

iii) $\left(\frac{x^{a^2+b^2}}{x^{ab}}\right)^{a+b}\left(\frac{x^{b^2+c^2}}{x^{bc}}\right)^{b+c}\left(\frac{x^{c^2+a^2}}{x^{ac}}\right)^{a+c} = x^{2(a^3+b^3+c^3)}$

(Hint :- use identity $a^3+b^3 = (a+b)(a^2+b^2-ab)$)

iv.) $\left\{\left(x^{a-a^{-1}}\right)^{\frac{1}{a-1}}\right\}^{\frac{a}{a+1}} = x$

v.) $\left(\frac{a^{x+1}}{a^{y+1}}\right)^{x+y}\left(\frac{a^{y+2}}{a^{z+2}}\right)^{y+z}\left(\frac{a^{z+3}}{a^{x+3}}\right)^{z+x} = 1$

vi.) $\left(\frac{3^a}{3^b}\right)^{a+b}\left(\frac{3^b}{3^c}\right)^{b+c}\left(\frac{3^c}{3^a}\right)^{c+a} = 1$

vii) $\frac{\left(a+\frac{1}{b}\right)^m \times \left(a-\frac{1}{b}\right)^n}{\left(b+\frac{1}{a}\right)^m \times \left(b-\frac{1}{a}\right)^n} = \left(\frac{a}{b}\right)^{m+n}$

viii) $\sqrt[lm]{\frac{x^l}{x^m}} \times \sqrt[mn]{\frac{x^m}{x^n}} \times \sqrt[nl]{\frac{x^n}{x^l}} = 1$

ix.) $\left(\frac{x^{a+b}}{x^c}\right)^{a-b}\left(\frac{x^{b+c}}{x^a}\right)^{b-c}\left(\frac{x^{c+a}}{x^b}\right)^{c-a} = 1$

---

x.) $\left\{(x^a)^b\right\}^{\frac{1}{ab}}\left\{(x^b)^c\right\}^{\frac{1}{bc}}\left\{(x^c)^a\right\}^{\frac{1}{ac}} = 1$

TYPE-X PROVING QUESTIONS (HOTS)

Q38.) if $a^x = b$, $b^y = c$ and $c^z = a$ prove that $xyz = 1$.

→ we know that

$$a^{xyz} = (a^x)^{yz} \quad [\because a^x = b]$$

$$a^{xyz} = (b)^{yz}$$

$$a^{xyz} = (b^y)^z \quad [\text{as } b^y = c)$$

$$a^{xyz} = c^z$$

$$a^{xyz} = a \quad (\text{as } c^z = a)$$

$\therefore$ on comparing $xyz = 1$

Q39.) if $a^x = b^y = c^z$ & $b^2 = ac$, prove that $y = \dfrac{2xz}{x+z}$

→ Let $a^x = b^y = c^z = k$, then

$$a = k^{\frac{1}{x}},\ b = k^{\frac{1}{y}},\ c = k^{\frac{1}{z}}$$

Now $b^2 = ac$

$$\Rightarrow (k^{1/y})^2 = k^{1/x} \times k^{1/z}$$

$$k^{2/y} = k^{\frac{1}{x} + \frac{1}{z}}$$

$$\Rightarrow \frac{2}{y} = \frac{1}{x} + \frac{1}{z} \Rightarrow \frac{2}{y} = \frac{x+z}{xz} \Rightarrow \boxed{y = \frac{2xz}{x+z}}$$

Q40) i.) if $2^x = 3^y = 12^z$ show that $\frac{1}{z} = \frac{1}{y} + \frac{2}{x}$.

ii.) if $3^x = 5^y = (75)^z$, show that $z = \frac{xy}{2x+y}$

iii.) $x = 2^{1/3} + 2^{2/3}$ show that $x^3 - 6x = 6$

Q41) Simplify or prove that

i.) $\left(\sqrt{3 \times 5^{-3}} \div \sqrt[3]{3^{-1}}\sqrt{5}\right) \times \sqrt[6]{3 \times 5^6} = \frac{3}{5}$

ii.) $9^{3/2} - 3 \times 5^0 - \left(\frac{1}{81}\right)^{-1/2} = 15$

iii.) $\left(\frac{1}{4}\right)^{-2} - 3 \times 8^{2/3} \times 4^0 + \left(\frac{9}{16}\right)^{-1/2} = \frac{16}{3}$

iv.) $\frac{2^{1/2} \times 3^{1/3} \times 4^{1/4}}{10^{-1/5} \times 5^{3/5}} \div \frac{3^{4/3} \times 5^{-7/5}}{4^{-3/5} \times 6} = 10$

v.) $\sqrt{\frac{1}{4}} + (0.01)^{-1/2} - (27)^{2/3} = \frac{3}{2}$

vi.) $\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n} = \frac{3}{2}$

vii) $\left(\frac{64}{125}\right)^{-2/3} + \frac{1}{\left(\frac{256}{625}\right)^{1/4}} + \left(\frac{\sqrt{25}}{\sqrt[3]{64}}\right)^0 = \frac{61}{16}$

viii) $\frac{3^{-3} \times 6^2 \times \sqrt{98}}{5^2 \times \sqrt[3]{1/125} \times (15)^{-4/3} \times 3^{1/3}} = 28\sqrt{2}$

TYPE-XI LAWS OF RATIONALISATION

Q42.) Simplify

i.) $\sqrt[5]{16} \times \sqrt[5]{2}$ (Ans → 2)

ii.) $\dfrac{\sqrt[4]{243}}{\sqrt[4]{3}}$ (Ans → 3)

iii.) $\sqrt[3]{4} \times \sqrt[3]{16}$ (Ans → 4)

iv.) $\dfrac{\sqrt[4]{1250}}{\sqrt[4]{2}}$ (Ans → 5)

v.) $(3+2\sqrt{2})(3-2\sqrt{2})$ (Ans → 1)

vi.) $(\sqrt{8}-\sqrt{2})(\sqrt{8}+\sqrt{2})$ (Ans → 6)

vii.) $(2\sqrt{5}+3\sqrt{2})^2$ (Ans → $38+12\sqrt{10}$)

Q43) Rationalise the denominator

i.) $\dfrac{2}{\sqrt{3}}$ $\left(\text{Ans} \to \dfrac{2\sqrt{3}}{3}\right)$

ii.) $\dfrac{1}{3+\sqrt{2}}$ $\left(\text{Ans} \to \dfrac{3-\sqrt{2}}{7}\right)$

iii) $\dfrac{5}{\sqrt{3}-\sqrt{5}}$ $\left(\text{Ans} \to -\dfrac{5}{2}(\sqrt{3}+\sqrt{5})\right)$

iv.) $\dfrac{1}{7+3\sqrt{2}}$ $\left(\text{Ans} \to \dfrac{7-3\sqrt{2}}{31}\right)$

v.) $\dfrac{2\sqrt{7}}{\sqrt{11}}$ $\left(\text{Ans} \to \dfrac{2}{11}\sqrt{77}\right)$

vi.) $\dfrac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}$ $\left(\text{Ans} \to \dfrac{18+2\sqrt{10}-4\sqrt{6}+3\sqrt{15}}{19}\right)$

vii.) $\dfrac{7+3\sqrt{5}}{7-3\sqrt{5}}$ $\left(\text{Ans} \to \dfrac{47+21\sqrt{5}}{2}\right)$

viii.) $\dfrac{\sqrt{3}+1}{2\sqrt{2}-\sqrt{3}}$ $\left(\dfrac{2\sqrt{6}+3+2\sqrt{2}+\sqrt{3}}{5}\right)$

ix.) $\dfrac{b^2}{\sqrt{a^2+b^2}+a}$ $\left(\text{Ans} \to \sqrt{a^2+b^2}-a\right)$

Q44) Find value to 3 decimal places, given that

$\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$, $\sqrt{10} = 3.162$ and $\sqrt{5} = 2.236$

(i.) $\dfrac{\sqrt{2}+1}{\sqrt{5}}$ (Ans→1.079)

(ii.) $\dfrac{2-\sqrt{3}}{\sqrt{3}}$ (Ans→0.154)

(iii.) $\dfrac{\sqrt{10}-\sqrt{5}}{\sqrt{2}}$ (Ans→0.655)

(iv.) $\dfrac{2+\sqrt{3}}{3}$ (Ans→1.2444)

(v.) $\dfrac{\sqrt{10}+\sqrt{15}}{\sqrt{2}}$ (Ans→4.975)

(vi.) $\dfrac{\sqrt{5}+1}{\sqrt{2}}$ (Ans 2.288)

(vii.) $\dfrac{2+\sqrt{3}}{3}$

Q45.) If both a and b are rational numbers, find the values of a and b in each of following

(i.) $\dfrac{\sqrt{3}-1}{\sqrt{3}+1} = a+b\sqrt{3}$

→ Rationalising

$$\frac{(\sqrt{3}-1)(\sqrt{3}+1)}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2-1} \Rightarrow \frac{(\sqrt{3})^2+1-2\sqrt{3}}{(\sqrt{3})^2-1}$$

$$\Rightarrow \frac{3+1-2\sqrt{3}}{3-1} \Rightarrow \frac{4-2\sqrt{3}}{2} \Rightarrow \boxed{2-\sqrt{3}}$$

Since $\dfrac{\sqrt{3}-1}{\sqrt{3}+1} = a+b\sqrt{3}$

$\therefore 2-\sqrt{3} = a+b\sqrt{3}$ ⇒ On comparing

$\boxed{a=2, \quad b=-1}$

Q45)

ii.) $\dfrac{3+\sqrt{7}}{3-\sqrt{7}} = a+b\sqrt{7}$ $(a=8, b=3)$

iii.) $\dfrac{5+2\sqrt{3}}{7+4\sqrt{3}} = a+b\sqrt{3}$ $(a=11, b=-6)$

iv.) $\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = a+b\sqrt{15}$ $(a=4, b=1)$

v.) $\dfrac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = a-b\sqrt{6}$ $(a=2, b=-5/6)$

vi.) $\dfrac{5+3\sqrt{3}}{7+4\sqrt{3}} = a+b\sqrt{3}$ $(a=-1, b=1)$

vii.) $\dfrac{4+3\sqrt{5}}{4-3\sqrt{5}} = a+b\sqrt{5}$ $\left(a=-\dfrac{61}{29}, b=-\dfrac{24}{29}\right)$

Q46) Simplify each of the following

i.) $\dfrac{3}{5-\sqrt{3}} + \dfrac{2}{5+\sqrt{3}}$ $\left(\text{Ans} \rightarrow \dfrac{25+\sqrt{3}}{22}\right)$

ii.) $\dfrac{4+\sqrt{5}}{4-\sqrt{5}} + \dfrac{4-\sqrt{5}}{4+\sqrt{5}}$ $\left(\text{Ans} \rightarrow \dfrac{42}{11}\right)$

iii.) $\dfrac{\sqrt{5}-2}{\sqrt{5}+2} - \dfrac{\sqrt{5}+2}{\sqrt{5}-2}$ $(\text{Ans} \rightarrow -8\sqrt{5})$

iv.) $\dfrac{1}{3-\sqrt{8}} - \dfrac{1}{\sqrt{8}-\sqrt{7}} + \dfrac{1}{\sqrt{7}-\sqrt{6}} - \dfrac{1}{\sqrt{6}-\sqrt{5}} + \dfrac{1}{\sqrt{5}-2} = 5$

v.) $\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \frac{1}{\sqrt{4}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{6}} + \frac{1}{\sqrt{6}+\sqrt{7}}$

$+ \frac{1}{\sqrt{7}+\sqrt{8}} + \frac{1}{\sqrt{8}+\sqrt{9}} = 2.$

vi.) $\frac{1}{2+\sqrt{3}} + \frac{2}{\sqrt{5}-\sqrt{3}} + \frac{1}{2-\sqrt{5}}$ (Ans → 0.)

vii.) $\frac{2}{\sqrt{5}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{2}} - \frac{3}{\sqrt{5}+\sqrt{2}}$ (Ans → 0.)

Q47) evaluate $\frac{15}{\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}}$, is being given that $\sqrt{5} = 2.236$ & $\sqrt{10} = 3.162$

(Hint: $\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}$

$\Rightarrow \sqrt{10} + 2\sqrt{5} + 2\sqrt{10} - \sqrt{5} - 4\sqrt{5}$

$\Rightarrow 3\sqrt{10} - 3\sqrt{5} \Rightarrow 3(\sqrt{10}-\sqrt{5})$) (Ans ↓ 5.398)

Q48.) Simplify, $\sqrt{2} = 1.4142$, $\sqrt{3} = 1.732$, $\sqrt{5} = 2.2360$ & $\sqrt{6} = 2.4495$ and $\sqrt{10} = 3.162$

i.) $\frac{3-\sqrt{5}}{3+2\sqrt{5}}$ (Ans → 0.1002)

ii.) $\frac{1+\sqrt{2}}{3-2\sqrt{2}}$ (14.0710)

iii.) $\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} + \frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}}$ (Ans → 11) iv.) $\dfrac{7+3\sqrt{5}}{3+\sqrt{5}} - \dfrac{7-3\sqrt{5}}{3-\sqrt{5}}$ (Ans → $\sqrt{5}$)

TYPE-XII TO FIND VALUE GIVEN EXPRESSIONS

Q49) If $x = 2+\sqrt{3}$, find value of $x^2 + \frac{1}{x^2}$

→ we know that

$$(a+b)^2 = a^2+b^2+2ab$$

put $a = x$, $b = \frac{1}{x}$

$$\Rightarrow \left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2} + 2\times x\times\frac{1}{x}$$

$$\Rightarrow \left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2} + 2$$

$$\Rightarrow x^2+\frac{1}{x^2} = \left(x+\frac{1}{x}\right)^2 - 2$$

Let $x = 2+\sqrt{3}$, ~~$\frac{1}{2+\sqrt{3}}$~~

Now $\dfrac{1}{x} = \dfrac{1}{2+\sqrt{3}} \times \dfrac{2-\sqrt{3}}{2-\sqrt{3}} = \dfrac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$

Now $x^2+\dfrac{1}{x^2} = \left(2+\sqrt{3}+2-\sqrt{3}\right)^2 - 2$

$$\boxed{x^2+\frac{1}{x^2} = (4)^2 - 2 = 16-2 = 14}$$

Q50) if $x = 3-2\sqrt{2}$ find $x^2+\frac{1}{x^2}$ (Ans → 34)

Q51) if $x = 1-\sqrt{2}$, find value of $\left(x-\frac{1}{x}\right)^3$ (Ans → 8)

Q52) if $x = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ & $y = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, find x^2+y^2. (Ans → 98)

Q53.) if $a = \frac{2-\sqrt{5}}{2+\sqrt{5}}$ and $b = \frac{2+\sqrt{5}}{2-\sqrt{5}}$, find a^2-b^2.
(Ans → $-144\sqrt{5}$)

Q54.) if $x = 2+\sqrt{3}$, find value of $x^3+\frac{1}{x^3}$ (Ans → 52)

Q55) if $x = 3+\sqrt{8}$, find value of $x^2+\frac{1}{x^2}$ (Ans → 34)

Q56.) hots if $x = \frac{1}{2-\sqrt{3}}$, find x^3-2x^2-7x+5 (Ans → 3)

Q57) Simplify

1.) $\sqrt{3+2\sqrt{2}}$

→ $\sqrt{2+1+2\sqrt{2}}$ ⇒ $\sqrt{(\sqrt{2})^2+(1)^2+2\times\sqrt{2}\times 1}$

$\because (a+b)^2 = a^2+b^2+2ab$ ∴ ⇒ $\sqrt{(\sqrt{2}+1)^2}$

2) $\sqrt{(\sqrt{2}+1)^2} = \left((\sqrt{2}+1)^2\right)^{\frac{1}{2}} \Rightarrow \boxed{\sqrt{2}+1}$ Ans

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ii.) $\sqrt{3-2\sqrt{2}}$ (Ans → $\sqrt{2}-1$)

iii.) $\sqrt{5+2\sqrt{6}}$ (Ans → $\sqrt{3}+\sqrt{2}$)

Q58.) Find value of

(i) if $x = \sqrt{2}+1$, then $x - \frac{1}{x}$ (Ans → 2)

ii.) if $x = 3+2\sqrt{2}$, find $\sqrt{x} - \frac{1}{\sqrt{x}}$, (Ans → 2)

iii.) if $x = \frac{2}{3+\sqrt{7}}$, then $(x-3)^2$ (Ans → 7)

iv.) if $x+\sqrt{15} = 4$, find $x + \frac{1}{x}$ (Ans → 8)

v.) if $x = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ and $y = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$, then

$x+y+xy =$ ____ ? (Ans 9)

vi.) if $x = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ & $y = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, then x^2+xy+y^2

____ ? (Ans → 99)

vii.) if $x = \sqrt[3]{2+\sqrt{3}}$ find $x^3 + \frac{1}{x^3}$ (Ans → 4)

viii.) $x = \sqrt{6}+\sqrt{5}$, $x^2+\frac{1}{x^2}-2$? (Ans → 20)

Q59) Short Questions

i.) positive square root of $7+\sqrt{48}$ ___ ? $(2+\sqrt{3})$

ii.) if $\sqrt{2}=1.4142$, then $\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$ is ___ ? (0.4142)

iii) if $\sqrt{13-a\sqrt{10}}=\sqrt{8}+\sqrt{5}$, find a ___ ? (Ans → -4)

iv.) Rationalisation factor of $\sqrt{5}-2$ (Ans → $2+\sqrt{5}$)

(HOTS) 'BOOSTER QUESTIONS' (SELF PRACTICE)

Q60.) if $x=\frac{1}{2-\sqrt{3}}$ find value of $\frac{1}{x}$ and x^{-1}?

Q61) if $x=1-\sqrt{2}$ find value of $x^2-\frac{1}{x^2}$?

Q62) if $x=\frac{3+\sqrt{5}}{2}$, find value of $x^2+\frac{1}{x^2}$?

Q63) if $x=3\sqrt{5}+2\sqrt{2}$ and $y=3\sqrt{5}-2\sqrt{2}$ find value of i.) $x+y$ ii.) $x-y$ iii.) x^2+y^2.

Q64) if $x=\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ and $y=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ find value of i.) $(x+y)^2$ ii.) $(x-y)^2$ iii.) x^2y+xy^2.

Q65) Solve for x

(i) $2^{5x} \div 2^{x} = \sqrt[4]{16}$ (ii) $(343)^{\frac{2}{x}} = 49$

(iii) $(-5)^{x-2} = \left(\frac{1}{8}\right)^{3}$ (iv) $(16)^{2x-1} = 4^{x-5}$

Q66) Simplify $\sqrt{2+\sqrt{3}} + \sqrt{2-\sqrt{3}}$, when it is given that $\sqrt{6} = 2.499$ (Ans→2.449)

Q67) if $x = \frac{2+\sqrt{3}}{2-\sqrt{3}} + \frac{2-\sqrt{3}}{2+\sqrt{3}} + \frac{\sqrt{3}-1}{\sqrt{3}x}$ find value of $x^2 + \left(\frac{53}{x}\right)^2$

Q68) Prove that if $a = 1+\sqrt{2}+\sqrt{3}$ & $b = 1+\sqrt{2}-\sqrt{3}$

Prove that $a^2+b^2-2a-2b = 8$.

Q69) if $x = \frac{1}{2-\sqrt{3}}$ find value of

$x^3 - 2x^2 - 7x + 5$

Q70) find value of $\left\{(23+2^2)^{\frac{2}{3}} + (130-19)^{\frac{1}{2}}\right\}^2$.

Q71) find value of x if $\frac{3^{5x} \times (81)^2 \times (56)}{3^{2x}} = 3^7$. find value of x.

Q72) if $\frac{x}{x^{1.5}} = 8x^{-1}$; find x

Q73) if $abc = 1$, show that:

$$\left(1+a+\frac{1}{b}\right)^{-1}+\left(1+b+\frac{1}{c}\right)^{-1}+\left(1+c+\frac{1}{a}\right)^{-1}=1$$

Q74) if $x = \frac{4\sqrt{3}}{2-\sqrt{2}} - \frac{30}{4\sqrt{3}-3\sqrt{2}} - \frac{3\sqrt{2}}{3+2\sqrt{3}}$ find value of $(x^4+x^2+3) + \frac{1}{(x^4+x^2+3)}$

Q75) if $x = \sqrt[3]{28}$ and $y = \sqrt[3]{27}$ find value of $x+y - \frac{1}{x^2+xy+y^2}$? (Ans → 6.)

Q76.) V.V.I if $x = \frac{1}{2-\sqrt{3}}$, evaluate x^2-4x+9 & hence evaluate $x^3-4x^2+9x+10$. (Ans → 8, $26+8\sqrt{3}$)

Q77) if $\frac{9^{m+1}\left(3^{-\frac{n}{2}}\right)^{-2}-(27)^n}{(3^m \times 2)^3} = \frac{1}{729}$ prove that $m-n=2$.

Q78) if $x = \frac{\sqrt{a+2b}+\sqrt{a-2b}}{\sqrt{a+2b}-\sqrt{a-2b}}$, prove that $bx^2-ax+b=0$.

Q79) if $x = 3$, find value of $\left(x^{\frac{1}{3}}+x^{-1/3}\right)\left(x^{2/3}+x^{-2/3}-1\right)$

CHAPTER-02

POLYNOMIALS

CHAPTER-2

POLYNOMIALS

Poly (Many) Mials (terms)

POLYNOMIALS:- Let x be a variable (literal), n be a positive integer and $a_0, a_1, a_2, ---, a_n$ be constants (real no.s). Then $a_n x_n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + a_1 x + a_0$ is known as polynomial in variable x.

eg:- $2x+7$, $3x^2+4x+2$, x^3+x^2+x+1 etc are polynomials in variable x.

HOW TO DENOTE A POLYNOMIAL?

i.e How to name a polynomial?

We use notations $f(x)$, $g(x)$, $h(x)$ etc. to denote a polynomial in variable x

eg:- $f(x) = 2x^3+7x^2-4x+15$

$g(x) = 3x^4+7x^2-5$

$p(y) = 2y^2-3y+4$ is a polynomial in variable y.

CONDITION FOR NOT BEING A POLYNOMIAL

eg:- $7x^3-2x^2+3\sqrt{x}-4$ is not a polynomial as the exponent of x in $3\sqrt{x}$ is not a positive integer it is fractional.

also $x^2-x+\frac{2}{x}$ is not a polynomial.

NOTE:- only positive integral powers of variable.

IMPORTANT TERMS RELATED TO POLYNOMIALS

coefficient
Exponent

$$5x^2 + 2y - 7$$

constant
variable
operators
Terms

1.) TERMS OF POLYNOMIAL

terms of the polynomial are the parts of the equation which are generally seperated by "+" or "-" signs. So, each part of a polynomial in an equation is a term.

for eg:- $2x^2 + 5x + 4$, the no. of terms will be 3.

2.) COEFFICIENTS (TUMHARE KITNE DOST HAI?)

a numerical or constant quantity placed before and Multiplying the variable in an algebric expression

eg:- $4x^2$ (coefficient of x^2 is 4)

$3x^4 - 7x^2 + 2x - 3$ (coefficient of $x^3 = 0$)

3.) DEGREE (SABSE TOP KI POSITION)

→ degree of polynomial is defined as the highest ~~degree~~ power of variable in the polynomial.

LEADING TERM - term with highest power of variable

eg:- $2x^2 + 7x + 3$

$2x^2$ — Leading term

3 — constant term → term with degree = 0.

POLYNOMIALS ON THE BASIS OF TERMS

MONOMIAL
→ expression containing only one term
eg:- $2x^2y$, $4x^3$ etc.

BINOMIAL
→ expression containing only two terms
eg:- $2x + 3y$, $2x^2 + x$ etc

TRINOMIAL
→ expression containing only three terms,
eg:- $x + y + z$
$2x^2 + x + 2$ etc.

NOTE:-
→ an expression with more than three terms are itself known as polynomials (many terms)

POLY → Many

MIALS → Terms

POLYNOMIALS ON THE BASIS OF DEGREE

CONSTANT POLYNOMIAL

↳ degree zero is called (Zero polynomial or) constant polynomial

eg:- $f(x) = 2$

$g(x) = 12$

$h(y) = \frac{3}{2}$

NOTE

↳ degree of zero polynomial i.e $f(x) = 0$ is not defined as

$f(x) = 0$

$g(x) = 0x$

$h(x) = 0x^2$ etc.

all are zero polynomial & we can't determine degree SO NOT DEFINED

LINEAR POLYNOMIAL

↳ a polynomial of degree one is called a linear polynomial.

eg:- $2x+2$ etc

in general $f(x) = ax+b$.

QUADRATIC POLYNOMIAL

a polynomial of degree two is called quadratic.

eg:- $2x^2 - 4x + 5$ etc

in general:- ax^2+bx+c

CUBIC POLYNOMIAL

a polynomial of degree 3 is called CUBIC POLYNOMIAL

eg:- $7x^3+4x-12$ etc.

in general ax^3+bx^2+cx+d

Bi-QUADRATIC POLYNOMIAL

fourth degree polynomial

eg:- $4x^4+x^2+1$ etc.

in general:- $ax^4+bx^3+cx^2+dx+e$

ZEROES (ROOTS) OF POLYNOMIAL

Consider the polynomial $f(x) = 2x^3 - 3x^2 + 4x - 2$

replace x by 2 everywhere in $f(x)$

$f(2) = 2 \times (2)^3 - 3 \times (2)^2 + 4 \times 2 - 2 = 16 - 12 + 8 - 2 = 10$

we say that value of $f(x)$ at $x = 2$ is 10

∴ **VALUE OF POLYNOMIAL** value of polynomial $f(x)$ at $x = \alpha$ is obtained by substituting $x = \alpha$ in the given polynomial and is denoted by $f(\alpha)$

Now consider the polynomial

$$f(x) = x^3 - 6x^2 + 11x - 6$$

value of $f(x)$ at $x = 1$ $(x = \alpha)$

$$f(1) = 1 - 6 + 11 - 6 = 0$$

So when $f(\alpha) = 0$ then α is called

ZERO (ROOT) OR SOLUTION OF POLYNOMIAL $f(x)$

NOTE:- An n^{th} degree polynomial can have at most n real roots

IMPORTANT

Finding a zero or root of polynomial $f(x)$ means solving polynomial equation $f(x) = 0$.

FINDING ZERO OF A LINEAR POLYNOMIAL

↳ $f(x) = ax+b$, $a \neq 0$ is a linear polynomial then it has only one root given by $f(x)=0$.

i.e., $ax+b=0 \Rightarrow ax=-b \Rightarrow x=\frac{-b}{a}$

for eg: $f(x) = 2x+3$

$\therefore f(x) = 0$

$\Rightarrow 2x+3=0 \Rightarrow 2x=-3 \Rightarrow \boxed{x=\frac{-3}{2}}$

is a zero of polynomial

FINDING ZERO OF A QUADRATIC POLYNOMIAL

↳ first write expression in descending order of degree.

Then use $\boxed{\text{MIDDLE TERM SPLITTING}}$

eg: $f(x) = ax^2+bx+c = 0$

$\Rightarrow f(x) = x^2-5x+6$

$\Rightarrow f(x) = 0$

$\Rightarrow x^2-5x+6=0$

Product of $a \times c$ = sum of b

$\underbrace{x^2-3x}_{\text{take } x \text{ common}} \underbrace{+2x+6}_{\text{take 2 common}} = 0$

$x(x-3) -2(x-3)$

$(x-2)(x-3)=0 \Rightarrow \boxed{x=2,3}$

NOTE :- Degree is 2 so zeroes are also 2

~~Finding~~

FINDING ZEROES OF CUBIC POLYNOMIAL

0) DIVISION OF POLYNOMIAL

LONG DIVISION METHOD

1.) write polynomial in descending order.

2.) check the highest power and divide the terms by the same.

3.) Use answer in step 2 as the division symbol

4.) Now subtract it and bring down the next term.

5.) Repeat 2 to 4 until you have no more terms to carry down.

6.) Note the final Answer, including remainder will be in fraction form.

for eg:-

$$
\begin{array}{r|l}
 & y^2 - y + 4 \\
\hline
y+2 & y^3 + y^2 + 2y + 3 \\
 & -(y^3 - 2y^2) \\
\hline
 & -y^2 + 2y + 3 \\
 & -(-y^2 - 2y) \\
\hline
 & 4y + 3 \\
 & -(4y + 8) \\
\hline
 & -5 \\
\hline
\end{array}
$$

Here the remainder is -5.

Now $p(y) = y^3 + y^2 + 2y + 3$ is divided by $y+2$

Now $p(-2) = (-2)^3 + (-2)^2 + 2(-2) + 3$

$= -8 + 4 - 4 + 3$

$\boxed{p(-2) = -5}$

NOTE:- it follows that remainder obtained when $p(x)$ is divided by $x-a$ is equal to $p(a)$ i.e value of $p(x)$ at $x = a$.

IInd EXAMPLE $p(x) = x^4 - 3x^2 + 2x + 5$ divided by $x-1$.

$x-1 \,\big)\, x^4 - 3x^2 + 2x + 5 \,\big($ quotient: x^3

$x^4 - x^3 \rightarrow$ it is a problem

(AS CHOCLATE CANNOT BE MIXED WITH LADOO!)

$\therefore$ $x-1 \,\big)\, x^4 + 0x^3 - 3x^2 + 2x + 5 \,\big($ quotient: $x^3 + x^2 - 2x$

$\cancel{x^4} - x^3$

$- \quad +$

$\cancel{x^3} - 3x^2 + 2x + 5$

$\cancel{x^3} - x^2$

$- \quad +$

$-\cancel{2x^2} + \cancel{2x} + 5$

$-\cancel{2x^2} + \cancel{2x}$

$+ \quad -$

5

$\boxed{\text{Rem} = 5}$

$p(1) = (1)^4 - 3(1)^2 + 2(1) + 5 = \boxed{5}$

DIVISION ALGORITHM

↳ Dividend = Divisor × Quotient + Remainder

eg:-
$$\begin{array}{r|l} & \ \ 4 \rightarrow \text{Quotient} \\ 2 & \ \ 8 \rightarrow \text{Dividend} \\ & -8 \\ \hline & \ \ 0 \rightarrow \text{Rem} \end{array}$$
Divisor = 2

eg:- $8 = 2 \times 4 + 0$

•) REMAINDER THEOREM

→ Let $p(x)$ be any polynomial of degree greater than or equal to one and a be any real number. if $p(x)$ is divided by $(x-a)$, then remainder is equal to $p(a)$.

Proof:) Let $q(x)$ be the Quotient and $r(x)$ be the remainder when $p(x)$ is divided by $(x-a)$, then $p(x) = (x-a)q(x) + r(x)$

where $r(x) = 0$ or $r(x) =$ constant

as degree of $r(x) <$ degree of $(x-a)$

Now let $r(x) = r$ (be any constant)

Then, $p(x) = (x-a)q(x) + r$

put $x = a$ in (ii), we get.

$p(a) = (a-a)\,q(a) + r$

$p(a) = 0 + r \Rightarrow \boxed{p(a) = r}$

This shows that remainder is $p(a)$ when $p(x)$ is divided by $(x-a)$.

— x — x — x —

eg:- when $f(x) = 2x^4 - 6x^3 + 2x^2 - x + 2$ is divided by $x+2$. (i.e $x+2$ $\rightarrow x-(-2)$) (compare with $x-a$)

then remainder is given by

$f(-2) = 2(-2)^4 - 6(-2)^3 + 2(-2)^2 - (-2) + 2$

$= 2\times16 - 6\times(-8) + 2\times4 + 2 + 2$

$= 32 + 48 + 8 + 2 + 2 = 92$

$\therefore$ remainder is 92

— x — x — x —

• FACTOR THEOREM

→ Let $p(x)$ be a polynomial of a degree greater than or equal to 1 and a be a real no. such that $p(a) = 0$

i.) $(x-a)$ is a factor of $p(x)$

ii.) conversely, if $(x-a)$ is a factor of $p(x)$, then $p(a) = 0$.

IMPORTANT IDENTITIES

1.) $(a+b)^2 = a^2+b^2+2ab$

2.) $(a-b)^2 = a^2+b^2-2ab$

3.) $(a+b)^2+(a-b)^2 = 2(a^2+b^2)$

4.) $(a+b)^2-(a-b)^2 = 4ab$

Formed by expanding LHS using ① & ②.

5.) $(a+b)(a-b) = a^2-b^2$

6.) $(a+b)^3 = a^3+b^3+3ab(a+b)$

$= a^3+b^3+3a^2b+3ab^2$

7.) $(a-b)^3 = a^3-b^3-3ab(a-b)$

$= a^3-b^3-3a^2b+3ab^2$

8.) $a^3+b^3 = (a+b)(a^2-ab+b^2)$

9.) $a^3-b^3 = (a-b)(a^2+ab+b^2)$

10.) $(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$

$= a^2+b^2+c^2+2ab+2bc+2ca$

11.) $a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

12.) if $a+b+c=0$ then $a^3+b^3+c^3 = 3abc$.

TYPE-1 CLASSIFICATIONS OF POLYNOMIALS

Q1.) which are polynomials & which are not? (write Reasons)

i.) $3x^2 - 4x + 15$ (Yes)

ii.) $y^2 + 2\sqrt{3}$ (yes

iii.) $3\sqrt{x} + \sqrt{2}x$ (No)

iv.) $x - \frac{4}{x}$ (No)

v.) $x^{12} + y^3 + t^{50}$ (Yes)

Q2) write the degree of each of following polynomials.

i.) $7x^3 + 4x^2 - 3x + 12$

ii.) $12 - x + 2x^3$

iii.) $5y - \sqrt{2}$

iv.) 7

v.) 0

ANSWERS
i.) 3 ii.) 3 iii.) 1
iv.) 0
v.) Not Defined
↳ Refer intro.

Q3) identify polynomial & give reasons also classify them as constant, linear, quadratic & cubic.

i.) $f(x) = 4x^3 - x^2 - 3x + 7$

ii.) $g(x) = 2x^3 - 3x^2 + \sqrt{x} - 1$

iii.) $p(x) = \frac{2}{3}x^2 - \frac{7}{4}x + 9$

iv.) $q(x) = 2x^2 - 3x + \frac{4}{x} + 2$

v.) $h(x) = x^4 - x^{\frac{3}{2}} + x - 1.$

Q4) identify polynomial in one-variable, two-variable etc.

i.) $x^2 - xy + 7y^2$

ii.) $x^2 - 2xt + 7t^2 - x + t$

iii.) $t^3 - 3t^2 + 4t + 5$

iv.) $xy + yz + zx$

(Ans i.) 2 ii.) 2 iii.) 1 iv.) 3)

TYPE-2 ON THE BASIS OF ZEROES OF POLYNOMIAL

Q5) if $f(x) = 2x^3 - 13x^2 + 17x + 12$, find

i.) $f(2)$ ii.) $f(-3)$ (i.) 10, ii.) -210)

Q6) Show that $x = 1$ is a root of the polynomial $2x^3 - 3x^2 + 7x - 6$.

→ Let $f(x) = 2x^3 - 3x^2 + 7x - 6$

then $f(1) = 2 \times (1)^3 - 3(1)^2 + 7(1) - 6$

$= 2 - 3 + 7 - 6 = 0$

Hence $x = 1$ is root of polynomial $f(x)$.

Q7) verify if the following are zeroes or not

i.) $f(x) = 3x + 1$, $x = -\frac{1}{3}$

ii.) $g(x) = 3x^2 - 2$, $x = \frac{2}{\sqrt{3}}, \frac{-2}{\sqrt{3}}$

iii.) $f(x) = x^3 - 6x^2 + 11x - 6$, $x = 1, 2, 3$

iv.) $f(x) = 5x - \pi$, $x = \frac{4}{5}$

Q8.) if $x = \frac{4}{3}$ is a root of the polynomial $f(x) = 6x^3 - 11x^2 + kx - 20$, find the value of k.

↳ we have,

$$f(x) = 6x^3 - 11x^2 + kx - 20$$

$$\therefore f\left(\frac{4}{3}\right) = 6\left(\frac{4}{3}\right)^3 - 11\times\left(\frac{4}{3}\right)^2 + k\times\left(\frac{4}{3}\right) - 20$$

$$f\left(\frac{4}{3}\right) = \frac{128}{9} - \frac{176}{9} + \frac{4k}{3} - 20 \Rightarrow \frac{4k}{3} - \frac{76}{3}$$

$$\Rightarrow f\left(\frac{4}{3}\right) = \frac{4k - 76}{3}$$

Now $x = \frac{4}{3}$ is a root of polynomial $f(x)$

$$\therefore f\left(\frac{4}{3}\right) = 0 \Rightarrow \frac{4k - 76}{3} = 0$$

$$\Rightarrow 4k - 76 = 0 \Rightarrow 4k = 76$$

$$\Rightarrow \boxed{k = 19} \text{ Ans.}$$

Q9) if $x = 2$ is a root of the polynomial $f(x) = 2x^2 - 3x + 7a$, find value of a. (Ans → $-\frac{2}{7}$)

Q10) if $x = -\frac{1}{2}$ is a zero of the polynomial $p(x) = 8x^3 - ax^2 - x + 2$, find value of a (Ans → 6)

Q11) Find the zeroes of polynomial

(i) $f(x) = x-5$ (5) (iii) $h(x) = 2x$ (0)

(ii) $g(x) = 2x+5$ $(-\frac{5}{2})$ (iv) $2x+1$ $(-\frac{1}{2})$

Q12) if $x=2$ and $x=0$ are roots of polynomial
HOTS $f(x) = 2x^3 - 5x^2 + ax + b$. find values of a & b.

↳ we have,

$f(x) = 2x^3 - 5x^2 + ax + b$

$\therefore f(2) = 2\times(2)^3 - 5\times(2)^2 + a\times 2 + b$

$f(2) = 16 - 20 + 2a + b$

$f(2) = 2a + b - 4$ —(1)

and $f(0) = 2\times 0 - 5\times 0 + a\times 0 + b$

$\therefore f(0) = b$ —(2)

Since $x=2$, $x=0$ are root of polynomial

$\therefore f(2) = 0$ and $f(0) = 0$

$\Rightarrow 2a + b - 4 = 0$ and $\boxed{b = 0}$ (using (1) & (2))

$\Rightarrow 2a + b = 4$ and $\boxed{b = 0}$

$\Rightarrow 2a + 0 = 4$

$\Rightarrow 2a = 4$

$\Rightarrow \boxed{a = 2}$

Q13.) Find roots of the

i.) $p(x) = 2x^3 - 11x + 3x^2 - 6$ (Ans → 2, -3, $-\frac{1}{2}$)

ii.) $p(x) = x^3 - 6x^2 + 11x - 6$ (Ans → 1, 2, 3)

iii.) $f(x) = 2x^3 + x^2 - 7x - 6$ (2, $-\frac{3}{2}$, -1)

TYPE-3 REMAINDER THEOREM

Q14.) Find the remainder when the polynomial $p(x) = x^4 - 3x^2 + 2x + 1$ is divided by $x-1$.

(Hint: Find $p(1)$) (Ans → 1)

Q15.) Find the remainder when $p(x) = x^3 - ax^2 + 6x - a$ is divided by $(x-a)$. (Ans → $p(a) = 5a$)

Q16) Find the remainder

i.) when $p(x) = 2x^4 - 6x^3 + 2x^2 - x + 2$ is divided by $x+2$ (Ans → 92)

ii.) $p(x) = 4x^3 - 12x^2 + 14x - 3$ is divided by $x - \frac{1}{2}$ (Ans → $\frac{3}{2}$)

iii.) $f(x) = x^3 - 6x^2 + 2x - 4$ is divided by $3x - 1$ (Ans → $-\frac{107}{27}$)

iv.) $f(x) = 9x^3 - 3x^2 + x - 5$ is divided by $x - \frac{2}{3}$ (Ans → -3)

v.) $f(x) = 4x^4 - 3x^3 - 2x^2 + x - 7$ divided by $x - 1$ (Ans → -7)

Q17.) If the polynomials $ax^3 + 4x^2 + 3x - 4$ and $x^3 - 4x + a$ leave the same remainder when divided by $(x-3)$, find the value a.

↳ soln Let $p(x)$, $q(x)$ be the given polynomial

Now when $p(x)$ and $q(x)$ are divided by $x-3$

By Remainder theorem remainders are $p(3)$ and $q(3)$ respectively.

ATQ.) $p(3) = q(3)$

$\Rightarrow a \times 3^3 + 4 \times 3^2 + 3 \times 3 - 4 = 3^3 - 4 \times 3 + a$

$\Rightarrow 27a + 36 + 9 - 4 = 27 - 12 + a$

$\Rightarrow 26a + 26 = 0 \Rightarrow 26a = -26$

$\Rightarrow \boxed{a = -1}$

Q18.) The polynomials $ax^3 + 3x^2 - 13$ and $2x^3 - 5x + a$ are divided by $x+2$. If remainder in each case is the same, find value of a.

(Hint :- Same as above) (Ans → $\frac{5}{9}$)

Q19) If the polynomial $2x^3 + ax^2 + 3x - 5$ and $x^3 + x^2 - 4x + a$ leaves same remainder when divided by $x-2$, find value of a. (Ans → $\frac{-13}{3}$)

Q20) if $f(x) = x^4 - 2x^3 + 3x^2 - ax + b$ is a polynomial such that when it is divided by $x-1$ and $x+1$, the remainders are 5 and 19 respectively.

HOTS

Find the remainder when $f(x)$ is divided by $x-2$.

(Problem here is $f(x)$ has a & b constants values of which are unknown first find a and b)

Sol^n) when $f(x)$ is divided by $x-1$ and $x+1$ remainders are 5 and 19 respectively.

$\therefore f(1) = 5$ and $f(-1) = 19$ (as given.)

$\Rightarrow 1^4 - 2 \times 1^3 + 3 \times 1^2 - a \times 1 + b = 5$ — (1)

also as $f(-1) = 19$

$\Rightarrow (-1)^4 - 2 \times (-1)^3 + 3 \times (-1)^2 - a \times (-1) + b = 19$ — (2)

simplifying (1) & (2)

(1) $\Rightarrow 1 - 2 + 3 - a + b = 5 \Rightarrow 2 - a + b = 5$

$\Downarrow$

$\boxed{-a + b = 3}$ — (3)

(2) $\Rightarrow 1 + 2 + 3 + a + b = 19 \Rightarrow 6 + a + b = 19$

$\Downarrow$

$\boxed{a + b = 13}$ — (4)

from (3) $\boxed{b = 3 + a}$ put in (4)

$\Rightarrow \quad a+b=13$

$\Rightarrow \quad a+3+a=13 \Rightarrow 2a+3=13$

$\Rightarrow 2a=13-3 \Rightarrow 2a=10 \Rightarrow a=\frac{\cancel{10}^{5}}{\cancel{2}}$

$\therefore \boxed{a=5}$

$\therefore \quad b=3+a$

$b=3+a=3+5=8$

$\therefore \boxed{b=8}$

$\therefore f(x)=x^4-2x^3+3x^2-5x+8$

Now when $f(x)$ is divided by $x-2$

then $f(2)=(2)^4-2(2)^3+3(2)^2-5(2)+8$

$=16-16+12-10+8$

$\boxed{f(2)=10}$

TYPE-4 FACTOR THEOREM

Q21) Check if $f(x)=4x^3+4x^2-x-1$ is a multiple of $2x+1$?

(hint :- if remainder obtained on dividing $f(x)$ by $2x+1$ is 0 then it is a Multiple hence find $f(-\frac{1}{2})$!

Q22) Show that $(x-3)$ is a factor of polynomial $x^3-3x^2+4x-12$.

(Hint :- if remainder is zero ∴ find $P(3)$)

Q23) Show that $(x-1)$ is a factor of $x^{10}-1$ and also for $x^{11}-1$.

Q24) Show that $x+1$ and $2x-3$ are factors of $2x^3-9x^2+x+12$?

Q25) Find the value of a, if $x-a$ is a factor of x^3-a^2x+x+2.

Soln.) Let $p(x)=x^3-a^2x+x+2$

By factor theorem $(x-a)$ is a factor of $p(x)$ iff $p(a)=0$.

Now $p(a)=0$

$\Rightarrow a^3-a^2\times a+a+2=0 \Rightarrow a^3-a^3+a+2=0$

$\Rightarrow a+2=0 \Rightarrow \boxed{a=-2}$

Hence, $(x-a)$ is a factor of given polynomial, if $\boxed{a=-2}$.

Q26.) Find value of k, if $x+3$ is a factor of $3x^2+kx+6$. (Ans → $k=11$)

Q27.) Determine value of a for which polynomial $2x^4-ax^3+4x^2+2x+1$ is divisible by $1-2x$.
(Ans → $a=25$)

Q28) For what values of a is $2x^3+ax^2+11x+a+3$ exactly divisible by $(2x-1)$? (Ans → $a=-7$)

Q29) Find value of a if $x+a$ is a factor.

i.) $x^3+ax^2-2x+a+4$

ii.) $x^4-a^2x^2+3x-a$.

i.) Let $f(x)=x^3+ax^2-2x+a+4$

Now $x+a$ is a factor of $f(x)$

$\Rightarrow$ $(x-(-a))$ is a factor of $f(x)$

$\Rightarrow$ $f(-a)=0$

$\Rightarrow$ $(-a)^3+a(-a)^2-2(-a)+a+4=0$

$\Rightarrow$ $-a^3+a^3+2a+a+4=0$

$\Rightarrow$ $3a+4=0$ $\Rightarrow$ $\boxed{a=-\frac{4}{3}}$

ii.) Same as above (Ans → $a=0$)

Q30) Find value of a

i.) is $(x-5)$ a factor of $x^3-3x^2+ax-10$ (Ans → -8

ii.) if $x+2$ is a factor of $5x^3-7x^2-ax-28$ (Ans → 48)

Q31) Find values of k, if $x-3$ is a factor of $k^2x^3 - kx^2 + 3kx - k$. (Ans → $0, 3\sqrt{3}$)

Q32) In each of polynomials find value of a, if $x-a$ is a factor of.

i.) $x^6 - ax^5 + x^4 - ax^3 + 3x - a + 2$ (Ans → -1)

ii.) $x^5 - a^2x^3 + 2x + a + 1$ (Ans → $-\frac{1}{3}$)

iii) $x^3 - 2ax^2 + ax - 1$ ($x-2$ is a factor) (Ans → $\frac{7}{6}$)

Q33) Show that $(x-2)$ is a factor of the polynomial $f(x) = 2x^3 - 3x^2 - 17x + 30$ and hence factorize $f(x)$

Sol^n) by factor theorem, $(x-2)$ will be a factor of $f(x)$, if $f(2)=0$.

we have, $f(2) = 2\times2^3 - 3\times2^2 - 17\times2 + 30$

$= 16 - 12 - 34 + 30 = 0$

$\therefore (x-2)$ is a factor of $f(x)$

Continued on Next Page →

Now divide $f(x)$ by $(x-2)$

$$
\begin{array}{r|l}
 & 2x^2 + x - 15 \\
\hline
x-2 & 2x^3 - 3x^2 - 17x + 30 \\
 & 2x^3 - 4x^2 \\
 & - \quad + \\
\hline
 & x^2 - 17x + 30 \\
 & x^2 - 2x \\
 & - \quad + \\
\hline
 & -15x + 30 \\
 & -15x + 30 \\
 & + \quad - \\
\hline
 & 0
\end{array}
$$

Now use Division Algorithm.

Dividend = Divisor X Quotient + Remainder

$= (x-2)(2x^2+x-15) + 0$

$= (x-2)(2x^2+6x-5x-15)$

$= (x-2)(2x(x+3)-5(x+3))$

$= (x-2)(2x-5)(x+3)$

Q34) Factorise (i.) $x^3-6x^2+11x-6$

(Ans $\Rightarrow$ $(x-1)(x-2)(x-3)$]

(ii.) $x^4+2x^3-13x^2-14x+24$

(iii.) $3x^3-x^2-3x+1$ (Ans $\rightarrow$ $(x-1)(x+1)(3x-1)$

(iv.) $x^3-10x^2-53x-42$ ($(x+1)(x+3)(x-14)$)

Q35) Factorize $9z^3 - 27z^2 - 100z + 300$ given that $(3z+10)$ is a factor of it.

(Ans → $(3z+10)(3z-10)(z-3)$)

Q36) Factorise $2x^4 + x^3 - 14x^2 - 19x - 6$

→ Factors of constant term are $\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}$

Now

$f(-1) = 2(-1)^4 + (-1)^3 \cancel{+} -14(-1)^2 - 19(-1) - 6$
$= 2 - 1 - 14 + 19 - 6$
$= 21 - 21 = 0$

& $f(-2) = 2(-2)^4 + (-2)^3 \cancel{+} -14(-2)^2 - 19(-2) - 6$
$= 32 - 8 - 56 + 38 - 6 = 0$

Now $(x+1)$ & $(x+2)$ are factors

$\therefore (x+1)(x+2) = x^2 + 3x + 2$ is a factor of $f(x)$

Now divide $f(x)$ by $x^2 + 3x + 2$ to get other factors.

$$2x^2 - 5x - 3$$

$$x^2+3x+2 \;\big)\; 2x^4 + x^3 - 14x^2 - 19x - 6$$

$$2x^4 + 6x^3 + 4x^2$$

$$- \quad - \quad -$$

$$-5x^3 - 18x^2 - 19x - 6$$

$$-5x^3 - 15x^2 - 10x$$

$$+ \quad + \quad +$$

$$-3x^2 - 9x - 6$$

$$-3x^2 - 9x - 6$$

$$+ \quad + \quad +$$

$$0$$

Apply Division algorithm.

Dividend = Div × Q + R

$$2x^4 + x^3 - 14x^2 - 19x - 6 = (x^2+3x+2) \times (2x^2-5x-3) + 0$$

$$= (x+1)(x+2)(2x^2-5x-3)$$

$$= (x+1)(x+2)(x-3)(2x+1)$$

TYPE-5 FACTORISE ALGEBRIC EXPRESSIONS

TIPS TO THINK BEFORE FACTORISING

1.) Factorise means to break given expression into two or more parts

2.) Firstly try to check identities if applicable to Factorise any given expression

3.) else make pairs and seperate common terms. or by splitting middle term.

FACTORISATION BY TAKING OUT COMMON FACTORS

Q37) Factorise each of the following

i.) $2p(a-b) + 3q(5a-5b) + 4r(2b-2a)$

Soln) (Since No such identity hence taking common)

$= 2p(a-b) + 3\times 5q(a-b) + 4\times 2\times r(b-a)$

$= 2p(a-b) + 15q(a-b) + 8r(b-a)$

$= 2p(a-b) + 15q(a-b) - 8r(a-b)$ taking -1 common for establishing symmetry

$= (a-b)(2p + 15q - 8r)$

taking $(a-b)$ common from each term

ii.) $ab(a^2+b^2-c^2) + bc(a^2+b^2-c^2) - ca(a^2+b^2-c^2)$

(Ans→ $(a^2+b^2-c^2)(ab+bc-ca)$)

iii) $x(x^2+y^2-z^2)+y(-x^2-y^2+z^2)-z(x^2+y^2-z^2)$

(Ans $\rightarrow (x^2+y^2-z^2)(x-y-z)$)

---

iv) $a^3x+a^2(x-y)-a(y+z)-z$

$\rightarrow a^3x+a^2x-a^2y-ay-az-z$

$\Rightarrow (a^3x+a^2x)-(a^2y+ay)-(az+z)$

(Note:- Make pairs in such a manner so that you can take out common values.)

$\Rightarrow a^2x(a+1)-ay(a+1)-z(a+1)$

$\Rightarrow (a+1)(a^2x-ay-z)$

(take $(a+1)$ common from each term)

---

v.) $(x^2+3x)^2-5(x^2+3x)-(y^2+3x)+5y$

(same as above ans $\rightarrow (x^2+3x-5)(x^2+3x-y)$)

---

vi) x^3+x-3x^2-3 (Ans $\rightarrow (x-3)(x^2+1)$)

vii) $a(a+b)^3-3a^2b(a+b)$ (Ans $\rightarrow a(a+b)(a^2+b^2-ab)$)

viii) $x(x^3-y^3)+3xy(x-y)$ (Ans $\rightarrow x(x-y)(x^2+y^2+xy+3y)$)

ix) $a^2x^2+(ax^2+1)x+a$ (Ans $\rightarrow (x+a)(ax^2+1)$)

x.) $x^2+y-xy-x$ (Ans $\rightarrow (x-1)(x-y)$)

xi) $x^3-2x^2y+3xy^2-6y^3$ (Ans $\rightarrow (x-2y)(x^2+3y^2)$)

xii.) $6ab-b^2+12ac-2bc$ (Ans $\rightarrow (6a-b)(b+2c)$)

xiii) $x(x-2)(x-4)+4x-8$ (Ans→ $(x-2)^3$)

xiv) $(a-b+c)^2+(b-c+a)^2+2(a-b+c)(b-c+a)$
(Ans→ $4a^2$)

xv) $a^2+2ab+b^2-c^2$ (Ans→ $(a+b+c)(a+b-c)$)

xvi) $(x+2)(x^2+25)-10x^2-20x$ (Ans→ $(x+2)(x-5)^2$)

FACTORISATION BY USING IDENTITIES

Q38) Factorise following expressions.

i.) $4a^2+12ab+9b^2-8a-12b$

↳ $(2a)^2+2\times 2a\times 3b+(3b)^2-4(2a+3b)$

(using $a^2+2ab+b^2=(a+b)^2$)

↳ $(2a+3b)^2-4(2a+3b)$

↳ $(2a+3b)(2a+3b-4)$

---

ii.) $a^2+b^2-2(ab-ac+bc)$

↳ $\underbrace{a^2+b^2-2ab}_{}+2ac-2bc$ (using $(a-b)^2 = a^2+b^2-2ab$)

$= (a-b)^2+2c(a-b)$

$= (a-b)(a-b+2c)$

---

iii) $\left(5x-\frac{1}{x}\right)^2+4\left(5x-\frac{1}{x}\right)+4$

$\hookrightarrow \left(5x - \frac{1}{x}\right)^2 + 2 \times \left(5x - \frac{1}{x}\right) \times 2 + 2^2$

$\hookrightarrow \left(5x - \frac{1}{x} + 2\right)^2 \quad (a^2 + b^2 + 2ab = (a+b)^2)$

iv) $4(x+y)^2 - 28y(x+y) + 49y^2$

(Ans $\rightarrow (2x + 2y - 7y)^2 \rightarrow (2x - 5y)^2$)

v.) $(2a+3b)^2 + 2(2a+3b)(2a-3b) + (2a-3b)^2$

(Ans $\rightarrow 16a^2$)

vi.) $x^8 - y^8$ (Hint $(x^4)^2 - (y^4)^2$)

(Ans $(x-y)(x+y)(x^2+y^2)(x^4+y^4)$)

vii.) $a^{12}x^4 - a^4x^{12}$ (take a^4x^4 common)

(Ans $\rightarrow a^4x^4(a^4+x^4)(a^2+x^2)(a+x)(a-x)$)

viii) $4a^2 - 9b^2 - 2a - 3b$

$\hookrightarrow (2a)^2 - (3b)^2 - 2a - 3b$ (using $a^2 - b^2 = (a-b)(a+b)$)

$\hookrightarrow (2a-3b)(2a+3b) - (2a+3b)$

$\hookrightarrow \boxed{(2a+3b)(2a-3b-1)}$ Ans

ix) $x^2 + 2xy + y^2 - a^2 + 2ab - b^2$

$\hookrightarrow$ (Hint : $x^2 + 2xy + y^2 - (a^2 - 2ab + b^2)$)

(Ans $\rightarrow (x+y+a-b)(x+y-a+b)$)

x) $3-12(a-b)^2$ (Ans $\rightarrow 3(1+2a-2b)(1-2a+2b)$)

xi) $x(x+z)-y(y+z)$ (Ans $\rightarrow (x-y)(x+y+z)$)
(Hint: Multiply and take common)

xii) a^2-b^2-a-b (Ans $\rightarrow (a+b)(a-b-1)$)

xiii) $25x^2-10x+1-36y^2$ (Ans $\rightarrow (1+a+b)(1-a-b)$)

xiv) $1-2ab-(a^2+b^2)$ (Hint: $1-(2ab+a^2+b^2)$)
(Ans $\rightarrow (1+a+b)(1-a-b)$)

xv) $a^2+4b^2-4ab-4c^2$ (Ans $\rightarrow (a-2b+2c)(a-2b-2c)$)

xvi) $\left(x^2+\frac{1}{x^2}\right)-4\left(x+\frac{1}{x}\right)+6$

Ans

$\hookrightarrow \left(\left(x+\frac{1}{x}\right)^2-2\right)-4\left(x+\frac{1}{x}\right)+6$ $\left\{\begin{array}{l}\text{using}\\ (a+b)^2=a^2+b^2+2ab\\ \Rightarrow a^2+b^2=(a+b)^2-2ab\end{array}\right\}$

$\hookrightarrow \left(x+\frac{1}{x}\right)^2-4\left(x+\frac{1}{x}\right)-2+6$

$\hookrightarrow \left(x+\frac{1}{x}\right)^2-4\left(x+\frac{1}{x}\right)+4$

$\hookrightarrow \left(x+\frac{1}{x}\right)^2-2\times 2\times\left(x+\frac{1}{x}\right)+(2)^2$

$\hookrightarrow \left(x+\frac{1}{x}-2\right)^2$ $\left(a^2-2ab+b^2=(a-b)^2\right)$

xvii) $4(x-y)^2-12(x-y)(x+y)+9(x+y)^2$

xviii) $xy^9 - yx^9$ (Ans → $-xy(x^2+y^2)(x+y)(x-y)(x^4+y^4)$)

xix) $x^4 + x^2y^2 + y^4$ (Ans → $(x^2+xy+y^2)(x^2-xy+y^2)$)

xx) $2(x+y)^2 - 9(x+y) - 5$ $(x+y-5)(2x+2y+1)$

hots

Q39) Factorise and simplify

i.) $x^4 + 4$ (SPECIAL TYPE)

(Since it is not a perfect square)

↳ $(x^4+4x^2+4)-4x^2$

↳ $(x^2+2)^2-(2x)^2$

↳ $(x^2+2-2x)(x^2+2+2x)$

↳ $\boxed{(x^2-2x+2)(x^2+2x+2)}$ Ans

ii.) x^4+4x^2+3

[Add and sub to make it perfect square]

↳ x^4+4x^2+4-1

↳ $(x^4+4x^2+4)-1$

↳ $(x^2+2)^2-(1)^2 \Rightarrow (x^2+2+1)(x^2+2-1)$

⇒ $\boxed{(x^2+3)(x^2+1)}$ Ans.

iii.) x^4+5x^2+9 (Ans → $(x^2-x+3)(x^2+x+3)$)

iv.) x^4+x^2+1 (Ans → $(x^2-x+1)(x^2+x+1)$)

FACTORISATION USING MIDDLE TERM SPLITTING

Q40) Factorise using appropriate Method.

i.) $25a^2 - 35a + 12$

(Since it does not fit into $a^2 + b^2 - 2ab$)

$\therefore$ using Middle term splitting

$\Rightarrow 25a^2 - 20a - 15a + 12$

$\Rightarrow (25a^2 - 20a) - (15a - 12)$

$\Rightarrow 5a(5a-4) - 3(5a-4) \Rightarrow \boxed{(5a-3)(5a-4)}$

ii) $9(x-2y)^2 - 4(x-2y) - 13$

Let $a = x - 2y$

$\Rightarrow 9a^2 - 4a - 13 \Rightarrow 9a^2 - 13a + 9a - 13$

$\Rightarrow (9a^2 - 13a) + (9a - 13)$

$\Rightarrow a(9a-13) + (9a-13)$

$\Rightarrow (a+1)(9a-13)$ (Now put value of a)

$\Rightarrow (x-2y+1)(9(x-2y)-13)$

$\Rightarrow \boxed{(x-2y+1)(9x-18y-13)}$ Ans.

NOTS

iii) $8(a+1)^2 + 2(a+1)(b+2) - 15(b+2)^2$

Put $(a+1) = x$ and $b+2 = y$

$\Rightarrow 8x^2 + 2xy - 15y^2$

$\Rightarrow 8x^2 + 12xy - 10xy - 15y^2$

$\Rightarrow (8x^2 + 12xy) - (10xy + 15y^2)$

$\Rightarrow 4x(2x+3y) - 5y(2x+3y)$

$\Rightarrow (2x+3y)(4x-5y)$

$\Rightarrow \left(2(a+1) + 3(b+2)\right)\left(4(a+1) - 5(b+2)\right)$

$\Rightarrow (2a+3b+8)(4a-5b-6)$

---

iv.) $7(x-2y)^2 - 25(x-2y) + 12$

(Ans $\rightarrow (x-2y-3)(7x-14y-4)$)

---

v.) $x^2 + 3\sqrt{3}x + 6$ (SPECIAL TYPE)

to factorize find two no.s such that their sum $= 3\sqrt{3}$ and product $= 6$

$\Rightarrow x^2 + 2\sqrt{3}x + \sqrt{3}x + 6$

$\Rightarrow x(x+2\sqrt{3}) + \sqrt{3}x + 2\sqrt{3}\times\sqrt{3}$

$\Rightarrow x(x+2\sqrt{3}) + \sqrt{3}(x+2\sqrt{3})$

$\Rightarrow \boxed{(x+\sqrt{3})(x+2\sqrt{3})}$ Ans.

---

vi) $x^2 + 3\sqrt{3}x - 30$

(Ans $\rightarrow (x+5\sqrt{3})(x-2\sqrt{3})$)

viii) $\underbrace{(x^2-4x)}_{a}\underbrace{(x^2-4x}_{a}\underbrace{-1}_{-1})-20$ (EXAM)

$\Rightarrow (x^2-4x)^2-(x^2-4x)-20$

$\Rightarrow a^2-a-20$ (where $a = x^2-4x$)

$\Rightarrow a^2-5a+4a-20$

$\Rightarrow a(a-5)+4(a-5) \Rightarrow (a+4)(a-5)$

$\Rightarrow (x^2-4x-5)(x^2-4x+4)$

$\Rightarrow (x^2-5x+x-5)(x^2-2\times x\times 2+2^2)$

$\Rightarrow (x(x-5)+x-5)(x-2)^2$

$\Rightarrow (x-5)(x+1)(x-2)^2$

---

ix) $4\sqrt{3}x^2+5x-2\sqrt{3}$

$\Rightarrow$ (find two numbers such that

Sum $= 5x$ and Product $= 4\sqrt{3}\times(-2\sqrt{3})x^2$
$= -24x^2$

$\Rightarrow 4\sqrt{3}x^2+8x-3x-2\sqrt{3}$

$\Rightarrow (4\sqrt{3}x^2+8x)-(3x+2\sqrt{3})$

$\Rightarrow 4x(\sqrt{3}x+2)-\sqrt{3}(\sqrt{3}x+2)$

$\boxed{\Rightarrow (\sqrt{3}x+2)(4x-\sqrt{3})}$ Ans.

x) $5\sqrt{5}x^2+30x+8\sqrt{5}$ (Ans → $\sqrt{5}(\sqrt{5}x+4)(\sqrt{5}x+2)$)

(Hint → $5\sqrt{5}x^2+10x+20x+8\sqrt{5}$)

xi) $7\sqrt{2}x^2-10x-4\sqrt{2}$ (Ans → $(x-\sqrt{2})(7\sqrt{2}x+4)$)

(Hint → $7\sqrt{2}x^2-14x+4x-4\sqrt{2}$)

		(Answers)
xii)	$x^2+6\sqrt{2}x+10$	$(x+5\sqrt{2})(x+\sqrt{2})$
xiii)	$x^2-2\sqrt{2}x-30$	$(x-5\sqrt{2})(x+3\sqrt{2})$
xiv)	$x^2-\sqrt{3}x-6$	$(x-2\sqrt{3})(x+\sqrt{3})$
xv)	$x^2+5\sqrt{5}x+30$	$(x+3\sqrt{5})(x+2\sqrt{5})$
xvi)	$x^2+2\sqrt{3}x-24$	$(x+4\sqrt{3})(x-2\sqrt{3})$
xvii)	$5\sqrt{5}x^2+20x+3\sqrt{5}$	$(5x+\sqrt{5})(\sqrt{5}x+3)$
xviii)	$2x^2+3\sqrt{5}x+5$	$(2x+\sqrt{5})(x+\sqrt{5})$

FACTORISATION USING IDENTITIES

Q41) Factorise the following

i) a^3+27

$\Rightarrow a^3+(3)^3$ (use $a^3+b^3=(a+b)(a^2-ab+b^2)$)

$\Rightarrow (a+3)(a^2-3a+9)$

ii.) $27a^3 + 125b^3$ (Ans → $(3a+5b)\{(3a)^2 - 3a\times 5b + (5b)^2\}$)

iii.) $(2a+1)^3 + (a-1)^3$ (Ans → $9a(a^2+a+1)$)

↳ hint use $a^3+b^3 = (a+b)(a^2-ab+b^2)$

---

iv.) $a^3 - 0.216$ (Ans → $(a-0.6)(a^2+0.6a+0.36)$)

v.) $p^6 - 512q^6$ (use $a^3-b^3 = (a-b)(a^2+ab+b^2)$)

$\Rightarrow (p^2)^3 - (8q^2)^3$

$\Rightarrow (p^2-8q^2)\{(p^2)^2 + p^2\times 8q^2 + (8q^2)^2\}$

$\Rightarrow (p^2-8q^2)(p^4+8p^2q^2+64q^4)$

---

v) $a^6 - b^6$ (Ans → $(a-b)(a+b)(a^2+ab+b^2)(a^2-ab+b^2)$)

(hint :- $(a^3)^2 - (b^3)^2 = \underbrace{(a^3-b^3)(a^3+b^3)}_{\text{Expand}}$)

---

vi) a^6+b^6 (Ans → $(a^2+b^2)(a^4-a^2b^2+b^4)$)

vii) a^7+ab^6 (Ans → $a(a^2+b^2)(a^4-a^2b^2+b^4)$)

↳ (hint :- $a(a^6+b^6)$ same as (v))

---

viii) $(x+1)^3 - (x-1)^3$ (Ans → $2(3x^2+1)$)

ix) $(x+1)^3 + (x-1)^3$ (Ans → $2x(x^2+3)$)

x) $8(x+y)^3 - 27(x-y)^3$ (Ans → $(-x+5y)(19x^2-10xy+7y^2)$)

xi) $x^{12}-y^{12}$

$\Rightarrow (x^6)^2-(y^6)^2$ (using $(a^2-b^2)=(a-b)(a+b)$)

$\Rightarrow (x^6-y^6)(x^6+y^6)$ (using $(a^3-b^3)=(a-b)(a^2+b^2+ab)$)

$\Rightarrow \left((x^2)^3-(y^2)^3\right)\left((x^2)^3+(y^2)^3\right)$ $a^3+b^3=(a+b)(a^2-ab+b^2)$

$\Rightarrow (x^2+y^2)\{(x^2)^2-x^2y^2+(y^2)^2\}\{(x^3+y^3)(x^3-y^3)\}$

$\Rightarrow (x^2+y^2)(x^4-x^2y^2+y^4)(x+y)(x^2-xy+y^2)(x-y)(x^2+xy+y^2)$

$\Rightarrow (x-y)(x+y)(x^2+y^2)(x^2+xy+y^2)(x^2-xy+y^2)(x^4+x^2y^2+y^4)$

---

xii) x^9-y^9 (Ans $\rightarrow (x-y)(x^2+xy+y^2)(x^6+x^3y^3+y^6)$)

Q42) Prove that

EXAM

$$\frac{0.87\times0.87\times0.87+0.13\times0.13\times0.13}{0.87\times0.87-0.87\times0.13+0.13\times0.13}=1$$

$$=\frac{(0.87)^3+(0.13)^3}{(0.87)^2-0.87\times0.13+(0.13)^2}$$

$$=\frac{a^3+b^3}{a^2-ab+b^2}$$ (where $a=0.87$ and $b=0.13$)

$$\frac{(a+b)(a^2-ab+b^2)}{(a^2-ab+b^2)}$$

$= (a+b) \Rightarrow (0.87+0.13) = \boxed{1}$

Hence proved

Q43.) Factorize (HOTS)

(i) x^3+3x^2+3x-7

$\Rightarrow x^3+3x^2+3x-7+1-1$ (Add & sub 1)

$\Rightarrow (x^3+3x^2+3x+1)-8$ $\quad (a+b)^3 = a^3+b^3+3ab(a+b)$

$\Rightarrow (x+1)^3-(2)^3$ $\quad (a^3-b^3 = (a-b)(a^2+ab+b^2))$

$\Rightarrow \{(x+1)-(2)\}\{(x+1)^2+2(x+1)+2^2\}$

$\Rightarrow (x-1)(x^2+2x+1+2x+2+4)$

$\Rightarrow \boxed{(x-1)(x^2+4x+7)}$ Ans

(ii) x^3-3x^2+3x+7

(iii) x^6-7x^3-8 (put $y=x^3$)

$\Rightarrow (x^3)^2-7x^3-8$

$\Rightarrow y^2-7y-8$

$\Rightarrow y^2-8y+y-8 \Rightarrow y(y-8)+(y-8)$

$\Rightarrow (y-8)(y+1) \Rightarrow (x^3-8)(x^3+1)$

$\Rightarrow (x-2)(x^2+2x+4)(x+1)(x^2-x+1)$ (Ans)

iv) $8x^3y^3+27a^3$ (Ans → $(2xy+3a)(4x^2y^2-6xya+9a^2)$)

v) $10x^4y-10xy^4$ (Ans → $(10xy)(x-y)(x^2+xy+y^2)$)

vi) $54x^6y+2x^3y^4$ (Ans → $2x^3y(3x+y)(9x^2-3xy+y^2)$)

vii) x^4y^4-xy (Ans → $(xy)(xy-1)(x^2y^2+xy+1)$)

viii) a^3+b^3+a+b (Ans → $(a+b)(a^2-ab+b^2+1)$)

IX) $a^3-\frac{1}{a^3}+\frac{2}{a}-2a$ (Ans → $\left(a-\frac{1}{a}\right)\left(a^2+\frac{1}{a^2}-1\right)$)

X) $a^{12}+b^{12}$

Q44) Simplify (EXAM)

i.) $\dfrac{155\times155\times155-55\times55\times55}{155\times155+155\times55+55\times55}$ (Ans → 100)

ii.) $\dfrac{1.2\times1.2\times1.2-0.2\times0.2\times0.2}{1.2\times1.2+1.2\times0.2+0.2\times0.2}$ (Ans → 1)

845) Factorize.

i.) $27a^3 + \frac{1}{64b^3} + \frac{27a^2}{4b} + \frac{9a}{16b^2}$

$\Rightarrow (3a)^3 + \frac{1}{(4b)^3} + \frac{9a}{4b}\left(3a + \frac{1}{4b}\right)$

$\Rightarrow (3a)^3 + \frac{1}{(4b)^3} + 3 \times 3a \times \frac{1}{4b}\left(3a + \frac{1}{4b}\right)$

$\Rightarrow \left(3a + \frac{1}{4b}\right)^3$ (Ans)

ii.) $\frac{64}{125}x^3 - 8 - \frac{96}{25}x^2 + \frac{48}{5}x$

$\Rightarrow \left(\frac{4x}{5}\right)^3 + (-2)^3 - \frac{24x}{5}\left(\frac{4x}{5} - 2\right)$

$\Rightarrow \left(\frac{4x}{5} - 2\right)^3$

iii.) $a^3 + 3a^2b + 3ab^2 + b^3 - 8$

(Ans $\rightarrow$ $(a+b-2)(a^2+b^2+2ab+2a+2b+4)$)

iv.) $8x^3 + 27y^3 + 36x^2y + 54xy^2$

v.) $x^3 - 12x(x-4) - 64$

Q46) Simplify (EXAM)

$$(x+y)^3-(x-y)^3-6y(x^2-y^2)$$

$\Rightarrow (x+y)^3-(x-y)^3-3\times 2y\times (x+y)(x-y)$

$\Rightarrow (x+y)^3-(x-y)^3-3\times\{(x+y)-(x-y)\}(x+y)(x-y)$

$(2y = (x+y)-(x-y))$

$\Rightarrow (x+y)^3-(x-y)^3-3(x+y)(x-y)((x+y)-(x-y))$

$\Rightarrow a^3-b^3-3ab(a-b)$ where $a = x+y$, $b = x-y$

$\Rightarrow (a-b)^3$

$\Rightarrow \{(x+y)-(x-y)\}^3 \Rightarrow (2y)^3 \Rightarrow \boxed{8y^3}$ Ans

Q47) factorize

i.) $8x^3+27y^3+z^3-18xyz$

$\Rightarrow (2x)^3+(3y)^3+z^3-3\times 2x\times 3y\times z$

$\Rightarrow (2x+3y+z)((2x)^2+(3y)^2+(z)^2-2x\times 3y-3y\times z-z\times 2x)$

$\Rightarrow (2x+3y+z)(4x^2+9y^2+6z^2-6xy-3yz-2xz)$

ii.) $a^3-8b^3-64c^3-24abc$

(Ans $\rightarrow (a-2b-4c)(a^2+4b^2+16c^2+2ab-8bc+4ca)$

iii) $a^3 - b^3 + 1 + 3ab$

$\Rightarrow a^3 + (-b)^3 + (1)^3 - 3(a)(-b)(1)$

$\Rightarrow (a-b+1)(a^2+b^2+1+ab-a+b)$

$\Rightarrow (a-b+1)(a^2+b^2+ab-a+b+1)$

---

iv) $2\sqrt{2}a^3 + 8b^3 - 27c^3 + 18\sqrt{2}abc$

(Hint:- $(\sqrt{2}a)^3 + (2b)^3 - (3c)^3 - 3 \times \sqrt{2}a \times 2b \times (-3c)$)

(Ans $\rightarrow (\sqrt{2}a+2b-3c)(2a^2+4b^2+9c^2-2\sqrt{2}ab+6bc+3\sqrt{2}ac)$)

---

v.) $2\sqrt{2}x^3 + 3\sqrt{3}y^3 + \sqrt{5}(5-3\sqrt{6}xy)$

HOTS

$\Rightarrow 2\sqrt{2}x^3 + 3\sqrt{3}y^3 + 5\sqrt{5} - 3 \times \sqrt{5} \times \sqrt{6}xy$

$\Rightarrow (\sqrt{2}x)^3 + (\sqrt{3}y)^3 + (\sqrt{5})^3 - 3 \times (\sqrt{2}x)(\sqrt{3}y)(\sqrt{5})$

$\Rightarrow$ (using $a^3+b^3+c^3-3abc$
$= (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$)

$\Rightarrow (\sqrt{2}x+\sqrt{3}y+\sqrt{5})(2x^2+3y^2+5-\sqrt{6}xy-\sqrt{15}y-\sqrt{10}x)$

Q48) Find the product

i.) $(a-b-c)(a^2+b^2+c^2+ab+ac-bc)$

ii.) $(3x-5y-4)(9x^2+25y^2+15xy+12x-20y+16)$

(Hint use identities)

i.) Ans $\rightarrow a^3-b^3-c^3-3abc$ ii.) $27x^3-125y^3-64-180xy$ Ans

Q49) Factorise

i) $(a+b)^3+(b+c)^3+(c+a)^3-3(a+b)(b+c)(c+a)$

$\Rightarrow$ (using $a^3+b^3+c^3-3abc$

$= (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$\rightarrow$ Expand $(a+b)^2 = a^2+b^2+2ab$

$\Rightarrow \{(a+b)+(b+c)+(c+a)\}\{(a+b)^2+(b+c)^2+(c+a)^2$

$-(a+b)(b+c)-(b+c)(c+a)-(c+a)(a+b)\}$

$\Rightarrow \{(2a+2b+2c)\}\{a^2+b^2+2ab+b^2+c^2+2ab+c^2+a^2+2ac$

$-(ab+ac+b^2+bc)-(bc+ba+c^2+ac)-(ca+cb$

$+a^2+ab)\}$

(open the brackets and simplify yourself)

$= 2(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

Q50) Prove that

$(a+b)^3+(b+c)^3+(c+a)^3-3(a+b)(b+c)(c+a)$

$= 2(a^3+b^3+c^3-3abc)$

(same as above)

Q51) Factorise

EXAM i) $(x-y)^3+(y-z)^3+(z-x)^3$

Let $x-y=a$, $y-z=b$ and $z-x=c$.

Then, $a+b+c = x-y+y-z+z-x = 0$

$$\therefore a^3+b^3+c^3 = 3abc$$

$$\Rightarrow (x-y)^3+(y-z)^3+(z-x)^3 = 3(x-y)(y-z)(z-x)$$

---

ii) $(x-2y)^3+(2y-3z)^3+(3z-x)^3$

(Ans $\rightarrow$ $3(x-2y)(2y-3z)(3z-x)$)

---

iii) $p^3(q-r)^3+q^3(r-p)^3+r^3(p-q)^3$

$\Rightarrow$ $(p(q-r))^3+(q(r-p))^3+(r(p-q))^3$

then same as above

Ans $\rightarrow$ $3pqr(p-q)(q-r)(r-p)$

---

iv) $(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3$

same as above

(Ans $\rightarrow$ $3(a+b)(b+c)(c+a)(a-b)(b-c)(c-a)$)

Q52.) Simplify: $\dfrac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3}$

(Exam)

Simplify Numerator

Since $a^2 - b^2 + b^2 - c^2 + c^2 - a^2 = 0$

$$\therefore (a^2-b^2)^3 + (b^2-c^2)^3 + (c^2-a^2)^3 = 3(a^2-b^2)(b^2-c^2)(c^2-a^2)$$

$$= 3(a-b)(a+b)(b-c)(b+c)(c-a)(c+a)$$

Now simplify denominator

$$(a-b) + (b-c) + (c-a) = 0$$

$$\Rightarrow (a-b)^3 + (b-c)^3 + (c-a)^3 = 3(a-b)(b-c)(c-a)$$

$$= \frac{\cancel{3}\cancel{(a-b)}(a+b)\cancel{(b-c)}(b+c)\cancel{(c-a)}(c+a)}{\cancel{3}\cancel{(a-b)}\cancel{(b-c)}\cancel{(c-a)}}$$

$$= (a+b)(b+c)(c+a)$$

Q53.) Find value of

$$(x-a)^3 + (x-b)^3 + (x-c)^3 - 3(x-a)(x-b)(x-c)$$

when $a+b+c = 3x$

$\Rightarrow$ here $(x-a) + (x-b) + (x-c)$

$$= 3x - (a+b+c)$$

$$= 3x - 3x = 0$$

$\therefore (x-a)^3 + (x-b)^3 + (x-c)^3 - 3(x-a)(x-b)(x-c) = 0$

Q54) Find value of $x^3 - 8y^3 - 36xy - 216$, when $x = 2y+6$, (EXAM)

$\Rightarrow x^3 - 8y^3 - 36xy - 216$

$\Rightarrow x^3 + (-2y)^3 + (-6)^3 - 3(x)(-2y)(-6)$

$\Rightarrow (x-2y-6)(x^2+4y^2+36+2xy-12y+6x)$

$= 0 \times (x^2+4y^2+36+2xy-12y+6x)$

$= 0$ [as $x = 2y+6 \Rightarrow x-2y-6=0$]

Q55) if $p = 2-a$, prove that $a^3 + 6ap + p^3 - 8 = 0$ (EXAM)

Q56) Find the value of $x^3 + y^3 - 12xy + 64$ when $x+y = -4$ (Ans $\rightarrow$ 0)

TYPE-6 ALGEBRIC IDENTIIES

1.) to expand expressions using identities
2.) to simplify products using identities
3.) to factorise using identities
4.) HOTS and most important Questions for exam.

Q57.) Expand each of the following — ANSWERS

i.) $(\sqrt{2}x-3y)^2$ — $(2x^2-6\sqrt{2}xy+9y^2)$

ii.) $(3x+4y)^2$ — $(9x^2+24xy+16y^2)$

iii.) $(x+5)(x-3)$ — $(x^2+2x-15)$

iv.) $\left(2x-\frac{1}{3x}\right)^2$ — $\left(4x^2-\frac{4}{3}+\frac{1}{9x^2}\right)$

v.) $(2x+3y)(2x-3y)$ — $(4x^2-9y^2)$

vi.) $(a-0.1)(a+0.1)$ — $(a^2-0.01)$

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vii.) $(1.5x^2-0.3y^2)(1.5x^2+0.3y^2)$ — $(2.25x^2-0.09y^4)$

viii.) $(x-2y-3z)^2$ — $(x^2+4y^2+9z^2-4xy+12yz-6zx)$

ix.) $(-x+2y+z)^2$ — $(x^2+4y^2+z^2-4xy+4yz-2zx)$

x.) $\left(\frac{a}{bc}+\frac{b}{ca}+\frac{c}{ab}\right)^2$ — $\left(\frac{a^2}{b^2c^2}+\frac{b^2}{c^2a^2}+\frac{c^2}{a^2b^2}+\frac{2}{a^2}+\frac{2}{b^2}+\frac{2}{c^2}\right)$

xi.) $\left(\frac{1}{3x}-\frac{2}{5y}\right)^2$ — $\left(\frac{1}{27x^3}-\frac{8}{125y^3}-\frac{2}{5xy}\left(\frac{1}{3x}-\frac{2}{5y}\right)\right)$

xii.) $(3x-2y)^3$ — $(27x^3-8y^3-54x^2y+36xy^2)$

Q58.) Find the products

i.) $(x-1)(x+1)(x^2+1)(x^4+1)$

(hint:- use $(a-b)(a+b) = a^2-b^2$))

$\Rightarrow (x^2-1)(x^2+1)(x^4+1)$

$\Rightarrow (x^4-1)(x^4+1)$ (same identity)

$\Rightarrow \boxed{(x^8-1)}$ Ans

---

ii.) $\left(x-\frac{1}{x}\right)\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}\right)\left(x^4+\frac{1}{x^4}\right)$

(hint:- use $(a-b)(a+b) = a^2-b^2$.)

$\Rightarrow \left(x^2-\frac{1}{x^2}\right)\left(x^2+\frac{1}{x^2}\right)\left(x^4+\frac{1}{x^4}\right)$

$\Rightarrow \left((x^2)^2-\left(\frac{1}{x^2}\right)^2\right)\left(x^4+\frac{1}{x^4}\right)$

$\Rightarrow \left(x^4-\frac{1}{x^4}\right)\left(x^4+\frac{1}{x^4}\right)$ (same identity)

$\Rightarrow \boxed{x^8-\frac{1}{x^8}}$

---

iii.) $(2x+y)(2x-y)(4x^2+y^2)$

$\Rightarrow ((2x)^2-(y)^2)(4x^2+y^2)$

$\Rightarrow (4x^2-y^2)(4x^2+y^2)$ (same identity)

$\Rightarrow ((4x^2)^2-(y^2)^2) \Rightarrow \boxed{16x^4-y^4}$ Ans iv.) $\left(x-\frac{y}{5}-1\right)\left(x+\frac{y}{5}+1\right)$

$\Rightarrow \left(x-\left(\frac{y}{5}+1\right)\right)\left(x+\left(\frac{y}{5}+1\right)\right)$ (use identity)

$\Rightarrow (x)^2-\left(\frac{y}{5}+1\right)^2$

$\Rightarrow x^2-\left(\frac{y^2}{25}+\frac{2y}{5}+1\right) \Rightarrow \left[x^2-\frac{y^2}{25}-\frac{2y}{5}-1\right]$ Ans.

v.) $\left(\frac{x}{2}-\frac{2}{5}\right)\left(\frac{2}{5}-\frac{x}{2}\right)-x^2+2x$ $\quad\left(-\frac{5}{4}x^2+\frac{12}{5}x-\frac{4}{25}\right)$

vi.) $(x^2+x-2)(x^2-x+2)$ $\quad(x^4-x^2+4x-4)$

vii.) $(x^3-3x^2-x)(x^2-3x+1)$ $\quad(x^5-x^2+4x-4)$

viii.) $(2x^4-4x^2+1)(2x^4-4x^2-1)$ $\quad(4x^8-16x^6+16x^4-1)$

IX.) $\left(m+\frac{n}{7}\right)^3\left(m-\frac{n}{7}\right)$ $\quad\left(m+\frac{n}{7}\right)^2\left(m^2-\frac{n^2}{49}\right)$

X.) $\left(\frac{1}{2}a-3b\right)\left(3b+\frac{a}{2}\right)\left(\frac{1}{4}a^2+9b^2\right)$ $\quad\left(\frac{1}{16}a^4-81b^4\right)$

XI.) $(x+3y)(x^2-3xy+9y^2)$ $\quad(x^3+27y^3)$

XII.) $(7a-5b)(49a^2+35ab+25b^2)$ $\quad(343a^3-125b^3)$

XIII.) $(0.9x+0.7y)(0.81x^2-0.63xy+0.49y^2)$ $\quad(0.729x^3+0.343y^3)$

xiv) $\left(\frac{2x}{5} - \frac{3y}{7}\right)\left(\frac{9y^2}{49} + \frac{4x^2}{25} + \frac{6xy}{35}\right)$ $\left(Ans \to \frac{8x^3}{125} - \frac{27y^3}{343}\right)$

xv) $7x^3+8y^3-(4x+3y)(16x^2-12xy+9y^2)$ $(-57x^3-19y^3)$

xvi) $(6m-n)(36m^2+6mn+n^2)-(3m+2n)^3$

$(Ans \to 189m^3-9n^3-54m^2n-36mn^2)$

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xvii) $(7p^4+q)(49p^8-7p^4q+q^2)$ $(Ans \to 343p^{12}+q^3)$

xviii) $(x^3+1)(x^6-x^3+1)$ $(Ans \to x^9+1)$

xix) $(x^2-1)(x^4+x^2+1)$ $(Ans \to x^6-1)$

xx) $\left(\frac{3}{x}-2x^2\right)\left(\frac{9}{x^2}+4x^4-6x\right)$ $\left(Ans \to \frac{27}{x^3}-8x^6\right)$

xxi) $\left(3+\frac{5}{x}\right)\left(9-\frac{15}{x}+\frac{25}{x^2}\right)$ $\left(Ans \to 27+\frac{125}{x^3}\right)$

---

xxii) $\left(\frac{3}{x}-2x^2\right)\left(\frac{9}{x^2}+4x^4-6x\right)$ $\left(Ans \to \frac{27}{x^3}-8x^6\right)$

xxiii) $(4x-3y+2z)(16x^2+9y^2+4z^2+12xy+6yz-8zx)$

$(Ans \to 64x^3-27y^3+8z^3+72xyz)$

xxiv) $(3x-4y+5z)(9x^2+16y^2+25z^2+12xy-15zx+20yz)$

$(Ans \to 27x^3-64y^3+125z^3+180xyz)$

Q59) Evaluate each of following by using identities

(i) 103×97

$\Rightarrow (100+3)(100-3)$ $(Ans \to 9991)$

ii.) 103×103 (Ans → 10609)

iii.) $(97)^2 \Rightarrow (100-3)^2$ (Ans → 9409)

iv.) $(0.99)^2 \Rightarrow (1-0.01)^2 \Rightarrow 0.9801$

v.) $185 \times 185 - 115 \times 115 \Rightarrow (21000)$

vi.) $0.54 \times 0.54 - 0.46 \times 0.46$ (Ans → 0.08)

vii.) $175 \times 175 + 2 \times 175 \times 25 + 25 \times 25$ (Ans → 40,000)

viii.) $322 \times 322 - 2 \times 322 \times 22 + 22 \times 22$ (Ans → 90,000)

ix.) $0.76 \times 0.76 + 2 \times 0.76 \times 0.24 + 0.24 + 0.24$ (Ans → 1)

x.) $\dfrac{7.83 \times 7.83 - 1.17 \times 1.17}{6.66}$ (Ans → 9)

860.) if $x + \frac{1}{x} = 6$, find

(i) $x^2 + \frac{1}{x^2}$ (Ans → 34)

(Hint $\left(x^2 + \frac{1}{x^2}\right) = \left(x + \frac{1}{x}\right)^2 - 2$)

↳ using $(a+b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow (a^2 + b^2 = (a+b)^2 - 2ab)$

ii.) $x^4+\frac{1}{x^4}$

$\Rightarrow$ Since $x^2+\frac{1}{x^2}=34$

Now $\left(x^2+\frac{1}{x^2}\right)^2=(34)^2$

$\Rightarrow (x^2)^2+\left(\frac{1}{x^2}\right)^2+2\times x^2\times\frac{1}{x^2}=1156$

$\Rightarrow x^4+\frac{1}{x^4}=1156-2$

$\Rightarrow \boxed{x^4+\frac{1}{x^4}=1154}$

Q61.) if $x^2+\frac{1}{x^2}=27$, find values of each of the following

a.) $x+\frac{1}{x}$ b.) $x-\frac{1}{x}$

a.) we have,

$\left(x+\frac{1}{x}\right)^2=x^2+\frac{1}{x^2}+2\times x\times\frac{1}{x}$

$\Rightarrow \left(x+\frac{1}{x}\right)^2=27+2$

$\left(x+\frac{1}{x}\right)^2=29 \Rightarrow \boxed{x+\frac{1}{x}=\pm\sqrt{29}}$

---

b.) $\left(x-\frac{1}{x}\right)^2=x^2+\frac{1}{x^2}-2 \Rightarrow 27-2=25$

$\therefore \boxed{x-\frac{1}{x}=\pm 5}$

Q62) if $x+y=12$ and $xy=32$, find value of x^2+y^2?

$\Rightarrow (x+y)^2 = x^2+y^2+2xy$

$\Rightarrow (12)^2 = x^2+y^2+2\times32 \Rightarrow x^2+y^2 = 144-64$

$\Rightarrow x^2+y^2 = 144-64$

$\Rightarrow x^2+y^2=80$

Q63) if $3x+2y=12$ and $xy=6$, find value of $9x^2+4y^2$?

$\Rightarrow$ (same as above) (Ans $\Rightarrow$ 72)

Q64) if $x^2+\frac{1}{x^2}=27$, find $x^3+\frac{1}{x^3}$?

Q65) if $x+\frac{1}{x}=11$ find $x^2+\frac{1}{x^2}$? (Ans $\rightarrow$ 119)

Q66) if $x-\frac{1}{x}=-1$, find $x^2+\frac{1}{x^2}$? (Ans $\rightarrow$ 3)

Q67) if $x+\frac{1}{x}=\sqrt{5}$, find values of $x^2+\frac{1}{x^2}$ and $x^4+\frac{1}{x^4}$? (Ans 3, 7)

Q68) if $4x^2+y^2=40$ and $xy=6$, find value of $2x+y$?
(Ans $\rightarrow \pm8$)

Q69.) if $x^2+\frac{1}{x^2}=66$ find $x-\frac{1}{x}$? (Ans ± 8)

Q70) Prove that

$$2a^2+2b^2+2c^2-2ab-2bc-2ca$$
$$=[(a-b)^2+(b-c)^2+(c-a)^2]$$

(hint:- grouping and use $(a+b)^2=a^2+b^2+2ab$)

$\Rightarrow a^2-2ab+b^2+a^2-2ca+c^2+b^2-2bc+c^2$

$\Rightarrow (a-b)^2+(b-c)^2+(c-a)^2$

(hence proved)

Q71) if $a^2+b^2+c^2-ab-bc-ca=0$, prove that $a=b=c$.

$\Rightarrow a^2+b^2+c^2-ab-bc-ca=0$

Multiply ~~and divide by~~ both sides by 2

$\Rightarrow 2(a^2+b^2+c^2-ab-bc-ca)=2\times 0$

$\Rightarrow 2a^2+2b^2+2c^2-2ab-2bc-2ca=0$

(same as above)

$\Rightarrow (a-b)^2+(b-c)^2+(c-a)^2=0$

$\Rightarrow a=b,\ b=c,\ c=a$ (since sum of squares is zero)

Q72.) if $a^2+b^2+c^2=20$ and $a+b+c=0$, find $ab+bc+ca$?

$\Rightarrow$ use

$\Rightarrow (a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

$\Rightarrow (a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$

$\Rightarrow (0)^2 = 20 + 2(ab+bc+ca)$

$\Rightarrow -20 = 2(ab+bc+ca)$

$\Rightarrow ab+bc+ca = \frac{-20}{2}$

$\Rightarrow \boxed{ab+bc+ca=-10}$

Q73.) if $a+b+c=9$ and $ab+bc+ca=40$, find $a^2+b^2+c^2$? (Ans $\rightarrow$ 1)

Q74) if $a^2+b^2+c^2=250$ and $ab+bc+ca=3$, find $a+b+c$? (Ans $\rightarrow \pm 16$)

Q75.) if $a+b+c=0$ and $a^2+b^2+c^2=16$, find value of $ab+bc+ca$? (Ans $\rightarrow -8$)

Q76) if $a^2+b^2+c^2=16$ and $ab+bc+ca=10$, find value of $a+b+c$? (Ans $\rightarrow \pm 6$)

Q77) if $a+b+c=9$ and $ab+bc+ca=23$, find the value of $a^2+b^2+c^2$? (Ans → 35)

Q78) if $x+y=12$ and $xy=27$, find value of x^3+y^3? (Ans → 756)

Q79) if $x-y=4$ and $xy=21$, find value of x^3-y^3? (Ans → 316)

Q80) if $x+\frac{1}{x}=7$, find value of $x^3+\frac{1}{x^3}$? (Ans → 322)

Q81) if $x-\frac{1}{x}=3$, find value of $x^3-\frac{1}{x^3}$? (Ans → 36)

Q82) if $2x+3y=13$ and $xy=6$ find value of $8x^3+27y^3$? (Ans → 793)

Q83) if $a+b=10$ and $a^2+b^2=58$, find value of a^3+b^3? (Ans → 370)

Q84) if $x^2+\frac{1}{x^2}=7$, find value of $x^3+\frac{1}{x^3}$?

(Hint: first find $x+\frac{1}{x}$ and the $x^3+\frac{1}{x^3}$)

(Ans → 18)

Q85) if $x^2+\frac{1}{x^2}=83$, find $x^3-\frac{1}{x^3}$? (Ans → 756)

Q86) If $x^4 + \frac{1}{x^4} = 47$, find value of $x^3 + \frac{1}{x^3}$?

Q87) Simplify

Ans

i.) $\left(\frac{x}{2} - \frac{2}{5}\right)\left(\frac{2}{5} - \frac{x}{2}\right) - x^2 + 2x$ $\left(-\frac{5}{4}x^2 + \frac{12}{5}x - \frac{4}{25}\right)$

ii.) $(x^2+x-2)(x^2-x+2)$ (x^4-x^2+4x-4)

iii.) $(x^3-3x^2-x)(x^2-3x+1)$ $(x^5-6x^4+9x^3-x)$

iv.) $(2x^4-4x^2+1)(2x^4-4x^2-1)$ $(4x^8-16x^6+16x^4-1)$

Q88) Simplify

i.) $(a+b+c)^2 + (a-b-c)^2$ $(2(a^2+b^2+c^2+2bc))$

ii.) $(a+b+c)^2 - (a-b-c)^2$ $(4a(b+c))$

iii.) $(2x+p-c)^2 - (2x-p+c)^2$ $(8x(p-c))$

iv.) $(x^2+y^2-z^2)^2 - (x^2-y^2+z^2)^2$ $4x^2(y^2-z^2)$

v.) $(x+y+z)^2 + \left(x + \frac{y}{2} + \frac{z}{3}\right)^2 - \left(\frac{x}{2} + \frac{y}{3} + \frac{z}{4}\right)^2$

vi.) $(x+y-2z)^2 - x^2 - y^2 - 3z^2 + 4xy$

(Ans → (v) $\frac{7x^2}{4} + \frac{41y^2}{36} + \frac{151z^3}{144} + \frac{8xy}{3} + \frac{13yz}{6} + \frac{29zx}{12}$

vi.) $z^2 + 6xy - 4yz - 4zx$

vii) $(x^2-x+1)^2-(x^2+x+1)^2$ (Ans → $-4x(x^2+1)$)

Q89) if $x+y=12$ and $xy=27$ find x^3+y^3

(Hint: $(a+b)^3=a^3+b^3+3ab(a+b)$)

(Ans → 756)

Q90) Simplify

i) $(4x+2y)^3+(4x-2y)^3$ — Ans ($128x^3+96xy^2$)

ii) $(4x+2y)^3-(4x-2y)^3$ — ($\cancel{128x^3+96xy^2}$) ($16y^3+192x^2y$)

Q91) if $x^2+\frac{1}{x^2}=98$, find $x^3+\frac{1}{x^3}$ (Ans → 970)

Q92) if $3x-2y=11$ and $xy=12$, find value of $27x^3-8y^3$ (Ans → 3707)

Q93) IMP if $x^4+\frac{1}{x^4}=119$ find $x^3-\frac{1}{x^3}$, $x^3+\frac{1}{x^3}$, $x+\frac{1}{x}$, $x^2+\frac{1}{x^2}$.

Q94) if $x-\frac{1}{x}=3+2\sqrt{2}$, find $x^3-\frac{1}{x^3}$

Q95) Simplify

i) $(x+3)^3 + (x-3)^3$ $\quad (2x^3+54x)$

ii) $\left(\frac{x}{2}+\frac{y}{3}\right)^3 - \left(\frac{x}{2}-\frac{y}{3}\right)^3$ $\quad \left(\frac{2y^3}{27}+\frac{x^2y}{2}\right)$

iii) $\left(x+\frac{2}{x}\right)^3 + \left(x-\frac{2}{x}\right)^3$ $\quad \left(2x^3+\frac{24}{x}\right)$

iv) $(x^3+1)(x^6-x^3+1)$ $\quad (x^9+1)$

v) $\left(\frac{3}{x}-2x^2\right)\left(\frac{9}{x^2}+4x^4-6x\right)$ $\quad \left(\frac{27}{x^3}-8x^6\right)$

Q96) evaluate without actually calculating the cubes

i) $(1.5)^3-(0.9)^3-(0.6)^3$ (Ans → 2.430)

ii) $(-12)^3+7^3+5^3$ (Ans → -1260)

iii) $(28)^3+(-15)^3+(-13)^3$ (Ans → 16380)

Q97) Find value

i) if $a+b+c=6$ and $ab+bc+ca=11$ find $a^3+b^3+c^3-3abc$ (Ans → 18) ii.) if $a+b+c=15$ and $a^2+b^2+c^2=83$ find $a^3+b^3+c^3-3abc$ (Ans → 180)

iii) if $x+y+z=1$, $xy+yz+zx=-1$ and $xyz=-1$ find value of $x^3+y^3+z^3$. (Ans → 2)

iv.) if $a+b+c=9$ and $a^2+b^2+c^2=35$, find $a^3+b^3+c^3-3abc$ (Ans → 108)

v.) if $a+b+c=9$ and $ab+bc+ca=26$, find $a^3+b^3+c^3-3abc$ (Ans → 27)

Q98.) (EXAM) Simplify: $\dfrac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3}$

(Ans → $(a+b)(b+c)(c+a)$)

Q99.) Find value

i.) if $x+\frac{1}{x}=3$, find $x^6+\frac{1}{x^6}$ (Ans → 322)

ii.) $x-\frac{1}{x}=\frac{1}{2}$, find $4x^2+\frac{4}{x^2}$ (Ans → 9)

iii.) $a^2+\frac{1}{a^2}=102$, find $a-\frac{1}{a}$ (Ans → 10)

Q100.) Mixed Questions

i.) if $\frac{a}{b} + \frac{b}{a} = -1$, find $a^3 - b^3$ (Ans 0)

ii.) ~~if~~ find $(x-y)(x+y)(x^2+y^2)(x^4+y^4)$

(Ans → $x^8 - y^8$)

iii.) if $3x + \frac{2}{x} = 7$ find $\left(9x^2 - \frac{4}{x^2}\right)$ (Ans → 35)

iv.) if $a+b+c=0$ then $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}$ (Ans → 3)

v.) if $a^{\frac{1}{3}} + b^{\frac{1}{3}} + c^{\frac{1}{3}} = 0$ (Ans → $(a+b+c)^3 = 27abc$)

vi.) $\frac{(a^2-b^2)^3 + (b^2-c^2)^3 + (c^2-a^2)^3}{(a-b)^3 + (b-c)^3 + (c-a)^3}$ (Ans → $(a+b)(b+c)(c+a)$)

vii.) product of $(a+b)(a-b)(a^2-ab+b^2)(a^2+ab+b^2)$ is equal to ______? (Ans → $a^6 - b^6$)

viii.) if $\frac{a}{b} + \frac{b}{a} = 1$, then $a^3+b^3 =$ ______? (Ans → 0)

ix.) if $49a^2 - b = \left(7a+\frac{1}{2}\right)\left(7a-\frac{1}{2}\right)$ find b? (Ans → $\frac{1}{4}$)

x.) if $(x^2-1)(x^4+x^2+1) =$ ______? ($x^6 - 1$)

CHAPTER-03

CO-ORDINATE GEOMETRY !!

CHAPTER-3

CO-ORDINATE GEOMETRY

INTRODUCTION

"CO-ORDINATE GEOMETRY IS LIKE LATITUDE AND LONGITUDE"

→ every place on this planet has coordinates that help us to locate it easily on the world map. The coordinate system of our earth is made up of imaginary lines called LATITUDE and LONGITUDES.

→ The zero degree 'GREENWICH LONGITUDE' and the zero degrees 'EQUATOR LATITUDE' are starting lines of this coordinate system.

→ Similarly locating the point in a plane or a piece of paper, we have the co-ordinate axes with horizontal 'X-AXIS' and the vertical 'Y-AXIS'

→ Co-ordinate geometry is the study of geometric figures by plotting them in the coordinate axes.

• WHAT IS COORDINATE GEOMETRY?

→ Co-ordinate geometry helps in presenting the geometric figures in two-dimensional plane and to learn properties of these figures.

→ here the position of point is defined using coordinate

~~AR coord~~

CO-ORDINATE PLANE

→ a cartesian plane divides the plane space into two dimensions and is useful to easily locate the points. It is also reffered as the COORDINATE PLANE.

→ Hence it is a 2D plane which is formed by the intersection of two perpendicular lines known as the x-axis and y-axis

WHAT ARE COORDINATES?

→ these are set of values which helps to show the exact position of a point in the coordinate plane.

→ location of any point on a plane is expressed by a pair of values (x, y) and these pairs are known as coordinates.

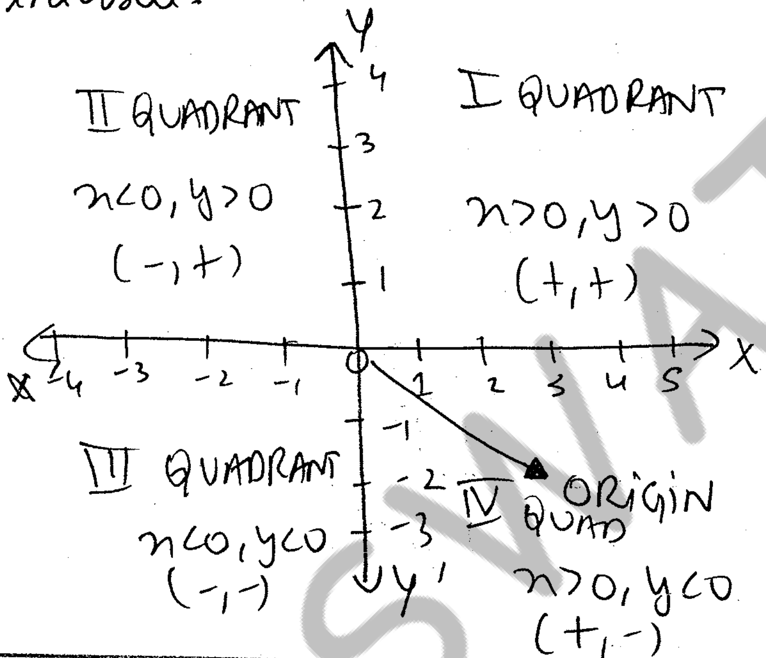
CARTESIAN CO-ORDINATE SYSTEM

i.) <u>CARTESIAN CO-ORDINATE AXES</u> :-

Let X'OX and Y'OY be two mutually perpendicular lines such that X'OX is HORIZONTAL LINE and YOY' is VERTICAL LINE in the same plane and intersect each other at O.

The line $X'OX$ called the $\boxed{\text{X-axis}}$ and the line YOY' is called $\boxed{\text{Y-axis}}$, and these two lines together are called the $\boxed{\text{CO-ORDINATE AXES}}$

The point 'O' is called the origin i.e the point at which axes intersect.



ii) QUADRANTS

→ co-ordinate axes $X'OX$ and YOY' divide the plane of graph paper in the four regions XOY, $X'OY'$, XOY, $X'OY$

These four regions are called QUADRANTS.

I QUADRANT – XOY – $x>0, y>0$ $(+,+)$

II QUADRANT – $X'OY$ – $x<0, y>0$ $(-,+)$

III QUADRANT – $X'OY'$ – $x<0, y<0$ $(-,-)$

IV QUADRANT – XOY' – $x>0, y<0$ $(+,-)$

iii.) CARTESIAN CO-ORDINATE OF A POINT

Let X'OX and Y'OY be the coordinate axes and let P be any point in the plane. to find the position of P with respect to X'OX and Y'OY.

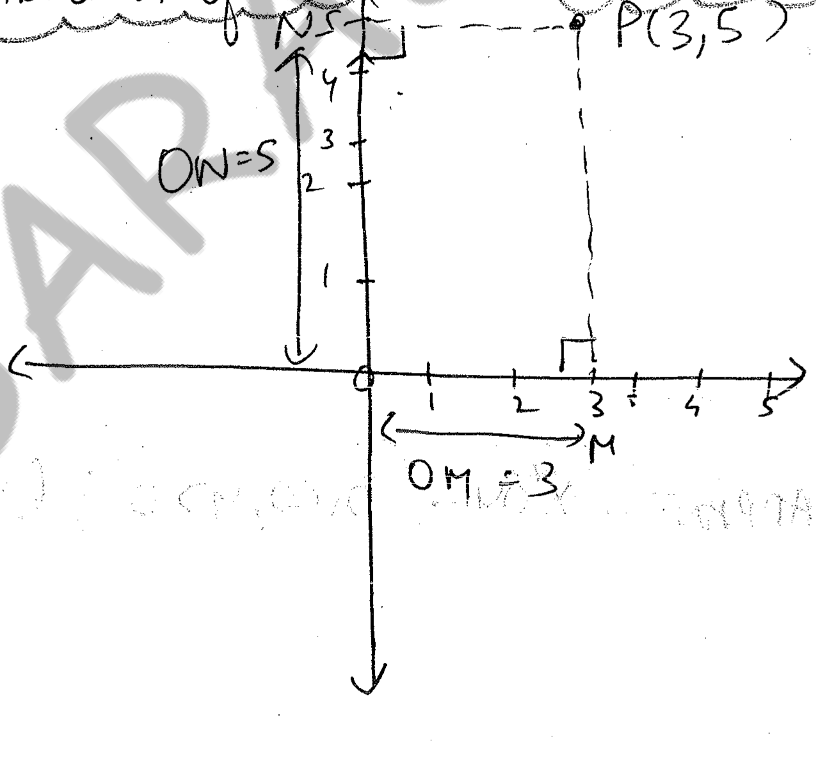
we draw perpendiculars from P on both co-ordinate axes. Let PM and PN be the perpendiculars on x-axis and y-axis respectively

Draw $PM \perp X'OX$ and $PN \perp Y'OY$

The length of line segment OM is called the x-coordinate or ABSUSSA of point P

The length of line segment ON is called Y-ORdinate or ORDINATE OF P

i.e ABSUSSA of P is 3 and ordinate is 5



iv.) POINTS ON THE AXES

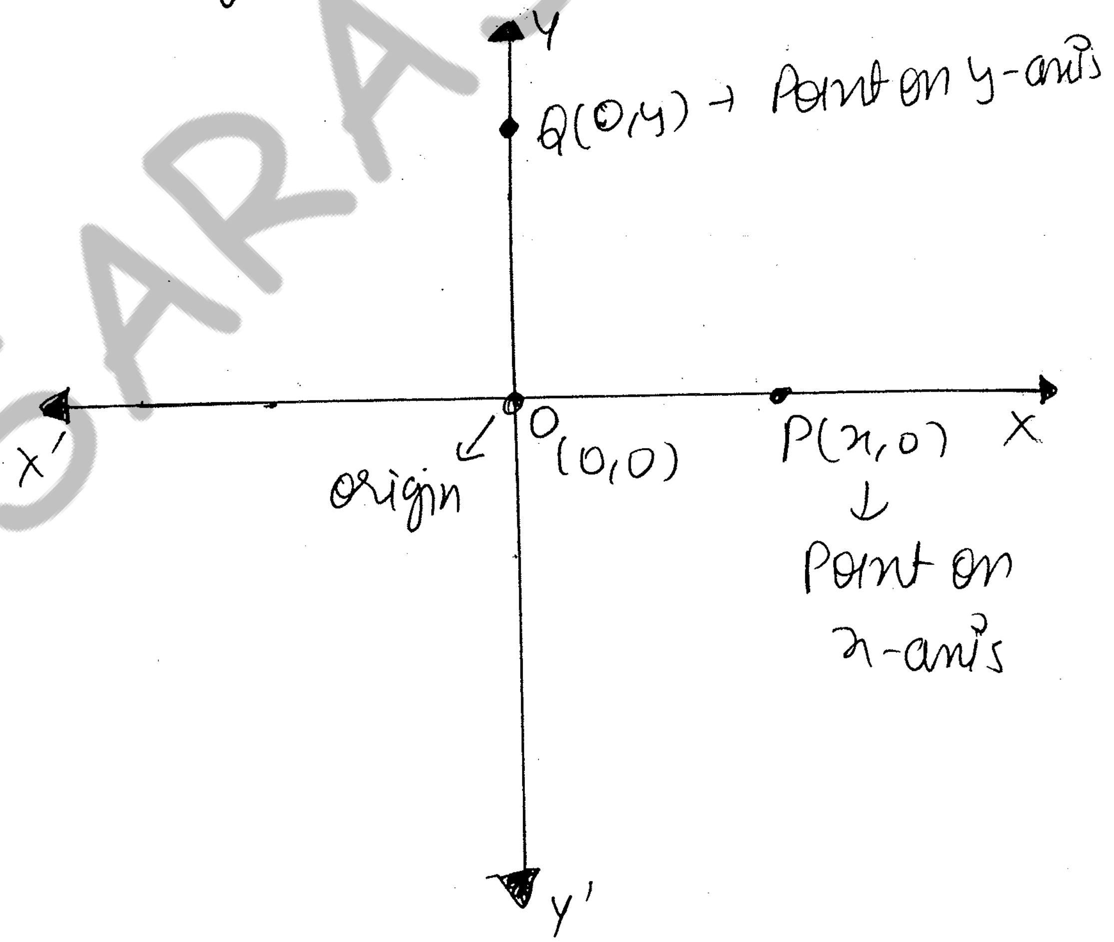
→ Point P lies on x-axis then clearly the distance of this point from x-axis is zero and ∴ ORDINATE OR y-co-ordinate of this point is zero

→ if any point lies on x-axis then its y-co-ordinate will be zero (x,0).

→ if we take a point on y-axis then if distance from y-axis is 0 and ~~tha~~ ∴ the ABSCISSA or x-coordinate of this point is zero.

→ if any point lies on y-axis then its x-co-ordinate will be (0,y).

→ co-ordinate of origin is zero. i.e O (0,0)



Q1) In which Quadrant do the following points lie

i.) (4,2)   ii.) (-3,5)   iii.) (-2,-5)   iv.) (4,-2)

v.) (-5,-5)   vi.) (-2,3)   vii.) (-5,3)   viii.) (4,-3)

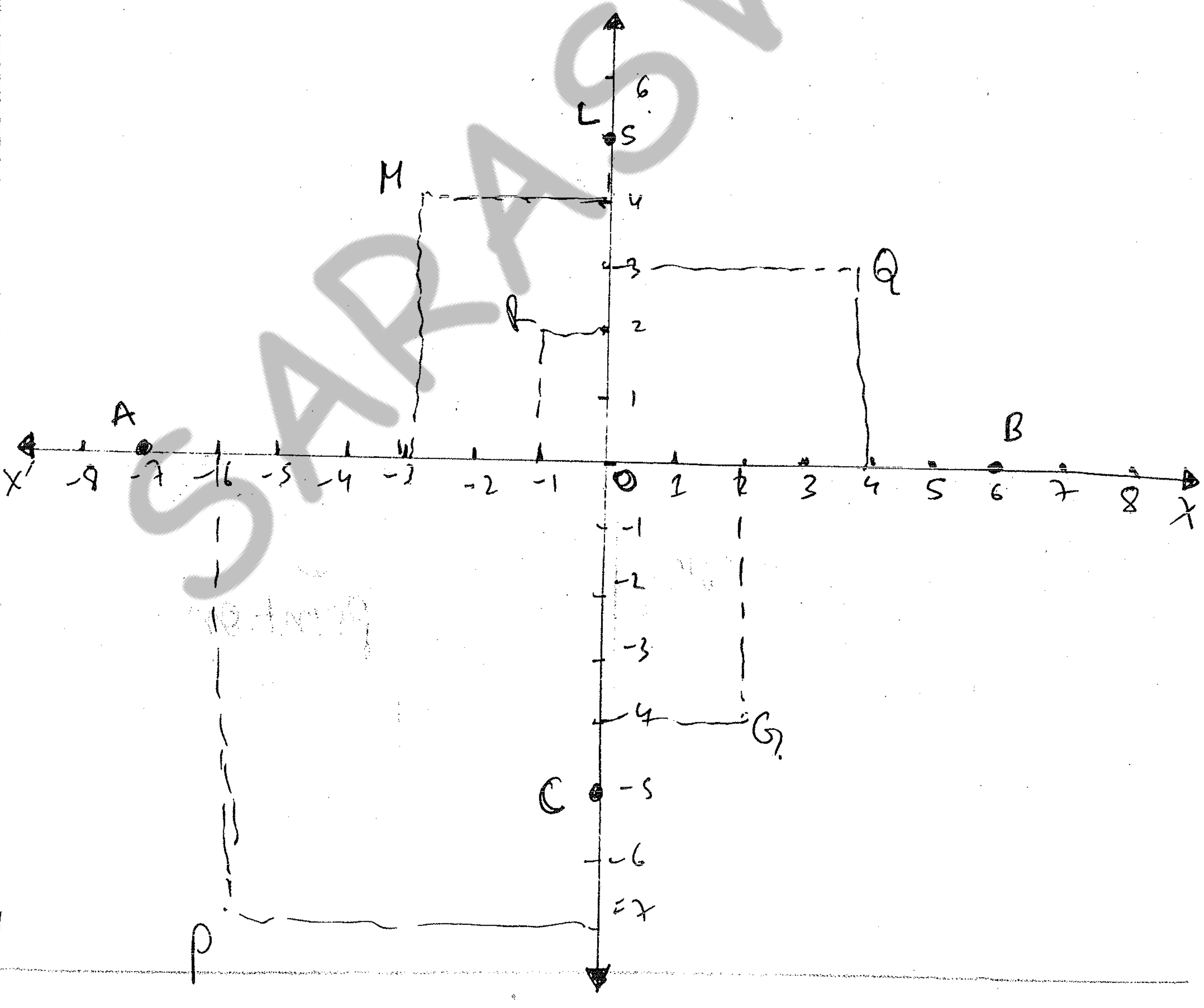
Q2) Plot the following on graph paper.

i.) (3,4)   ii.) (-2,3)   iii.) (-5,-2)   iv.) (4,-3)

v.) (3,0)   vi.) (-4,0)   vii.) (0,5)   viii.) (0,-7)

ix.) (-6,0)   x.) (0,0)

Q3) See the figure and answer the following



i.) write the coordinates of points A, B, L, C and O

ii.) point identified by co-ordinate $(-1, 2)$

iii.) coordinates of point P, M, G, Q?

iv.) ordinate of point B and L

v.) Absussa of A, C and G.

Q4.) Draw the Quadrilateral whose vertices are

i.) $(1,1)$, $(2,4)$, $(8,4)$ and $(10,1)$

ii.) $(-2,-2)$, $(-4,2)$, $(-6,-2)$ and $(-4,-6)$

Q5.) Plot the point $(-1,0)$, $(1,0)$, $(1,1)$, $(0,2)$, $(-1,1)$

Join them in given order. what figure do you get?

Q6.) Plot the point (x, y) using suitable units of distance

i.)

x	-2	-1	0	1	3
y	8	7	-1.25	3	-1

ii.)

x	-3	0	-1	4	2
y	7	-3.5	-3	4	-3

Q7.) Fill in the blanks

i.) Point of intersection of co-ordinate axes is ______

ii.) Abscissa and ordinate of origin are ______.

iii.) measure of angle between co-ordinate axes are ____.

iv.) A point whose Abscissa and ordinate are 2 & -5 lies in ________ Quadrant.

v.) Points (-4,0) and (7,0) lie on ____ axis.

vi.) ordinate of any point on x-axis is ______.

vii.) Abscissa of any point on y-axis is ______

viii) Abscissa of a point is positive in the ________ Quadrants.

ix) A point whose Abscissa is -3 and ordinate 2 lies in ______ Quadrant.

x.) two points having same abscissae but different ordinates lie on ________.

xi.) Perpendicular distance of point P(4,3) from x-axis is ______.

xii.) perpendicular distance of P(5,2) from y-axis is ______

XIII.) distance of P(4,3) from y-axis ______

XIV.) area of Δ formed by A(2,0), B(6,0) & C(4,6) is ______.

XV.) area of triangle formed by P(0,1), Q(0,5) and R(3,4) is ______.

CHAPTER - 12
HERON'S FORMULA

CHAPTER-12

HERON'S FORMULA

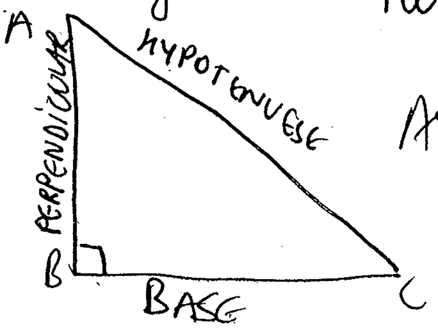
INTRODUCTION

- we are familiar with the shapes of many closed figures such as square, rectangles, Quadrilaterals, right triangles, equilateral triangles, isosceles triangle, Scalene triangles etc. we know some rules to find area and perimeter for above mentioned figures. But in this section we will mainly discuss areas of different types of triangles.

•) AREAS OF TRIANGLES USING BASE AND HEIGHT

AREAS OF RIGHT ANGLE TRIANGLE

→ when the triangle is right angles, we can directly apply the above mentione rule for base and heights by using two sides Containing the right angle as base and height.

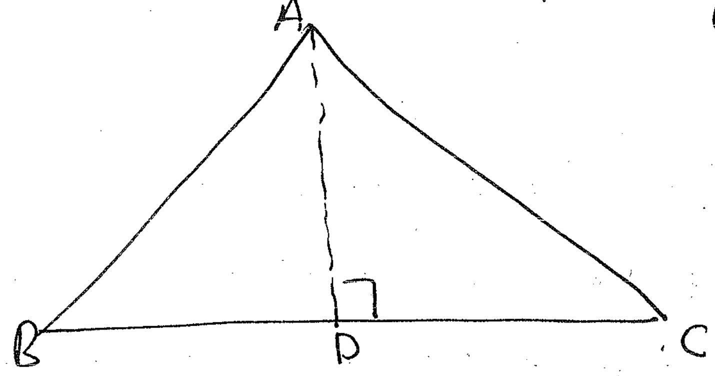


here base = BC , height = AB

Area of $\triangle ABC = \left(\frac{1}{2} \times AB \times BC\right)$ sq.unit

$= \frac{1}{2} \times base \times height$

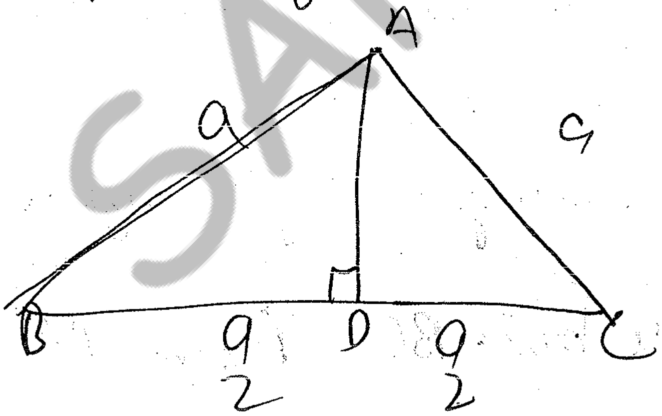
Also any side of triangle may be taken as base and the length of perpendicular from the opposite vertex to the base is the corresponding height.



here Area of $\triangle ABC = \frac{1}{2} \times \underset{\substack{\uparrow \\ \text{Base}}}{BC} \times \underset{\substack{\uparrow \\ \text{height}}}{AD}$

AREA OF AN EQUILATERAL TRIANGLE

Let ABC be equilateral triangle with side a and AD be the perpendicular from A on BC. Then D is the mid-point of BC i.e $BD = \frac{a}{2}$.



In right $\triangle ABD$

$AD^2 = AB^2 - BD^2$

(By Pythagoras)

$AD^2 = a^2 - \frac{a^2}{4} = \frac{3}{4}a^2$

$\Rightarrow \boxed{AD = \frac{\sqrt{3}}{2}a}$

so area of $\Delta ABC = \frac{1}{2} \times \underset{\text{Base}}{BC} \times \underset{\text{height}}{AD}$

$$= \frac{1}{2} \times a \times \frac{\sqrt{3}}{2} a = \frac{\sqrt{3}}{4} a^2 \text{ sq-units}$$

$\therefore$ Area of equilateral triangle is $= \left(\frac{\sqrt{3}}{4} a^2\right)$ sq-units

---

AREA OF AN ISOSCELES TRIANGLE

→ Let ABC be an isosceles triangle with $AB = AC = a$ and $BC = b$ and AD be the perpendicular from A on BC.

Then, D is the mid-point of BC i.e $BD = \frac{b}{2}$

In right-Angled ΔABD, by pythagoreas theorem

we have $AD^2 = AB^2 - BD^2$

$$\Rightarrow AD^2 = a^2 - \left(\frac{b}{2}\right)^2 = \frac{4a^2 - b^2}{4}$$

$$\boxed{AD = \frac{\sqrt{4a^2 - b^2}}{2}}$$

$\therefore$ Area of $\Delta ABC = \frac{1}{2} \times BC \times AD$

$$= \frac{1}{2} \times b \times \frac{\sqrt{4a^2 - b^2}}{2}$$

$$= \frac{1}{4} \times b \times \sqrt{4a^2 - b^2}$$

AREA OF TRIANGLE BY USING HERONS FORMULA

→ in a scalene triangle, if the length of each side is given but its height is not known and it cannot be obtained easily, we take the help of Heron's formula OR HERO'S FORMULA to find area of such triangle.

HERON'S FORMULA

if a, b, c denote the length of side of triangle ABC,

then area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

where $s = \dfrac{a+b+c}{2}$ is the semi-perimeter of ΔABC

REMARK

This formula is applicable to all types of triangles wether it is right angled, equilateral or isosceles.

Q1) Find the area of a triangle whose sides are 13cm, 14cm and 15cm.? (Ans→ $84cm^2$)

Q2) The perimeter of a triangular field is 450m and its sides are in the ratio 13:12:5. Find the area of triangle.

→ since a, b, c given sides are in Ratio 13:12:5

$\therefore a:b:c = 13:12:5$

$\Rightarrow a = 13x,\ b = 12x,\ c = 5x$

ATQ

Perimeter = 450

$\Rightarrow 13x + 12x + 5x = 450$

$\Rightarrow 30x = 450 \Rightarrow \boxed{x = 15}$

hence sides of triangles are

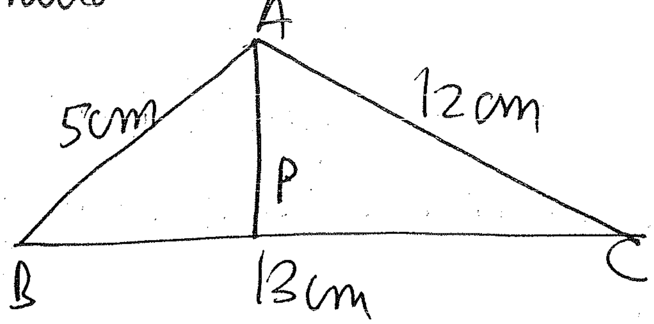
$a = 13 \times 15 = 195m,\ b = 12 \times 15 = 180m$

$c = 5 \times 15 = 75m.$

Now solve by applying Heron's Formula

Q3) The length of the sides of triangle are 5cm, 12cm and 13cm. Find the length of perpendicular from the opposite vertex to the side whose length is 13cm.

(Hint:- First find area of ΔABC by Heron's Formulla)



Here $A = 30cm^2$ (Area of ΔABC using heron's) — (1)

Now let p be the length of perpendicular from vertex A on the side BC.

Then Area of $\Delta ABC = \frac{1}{2} \times 13 \times p$ — (2)

from (1) & (2)

$$\frac{1}{2} \times 13 \times p = 30$$

$$\Rightarrow \boxed{p = \frac{60}{13} cm}$$

Q4.) A triangle and a parallelogram have the same base and the same area. if the sides of triangle are 26cm, 28cm and 30cm, and the parallelogram stands on the base 28 cm, find the height of parallelogram. (Ans → 12cm) (SAME AS NCERT 12.2)

Q5) Find the percentage increase in the area of a triangle if its each side is doubled.

→ Let a, b, c be the sides of old triangle and s be the semi-perimeter.

Then $s = \frac{1}{2}(a+b+c)$

The sides of new triangle are 2a, 2b and 2c

Let s' be the semi-perimeter. Then

$$s' = \frac{1}{2} \times (2a+2b+2c) = a+b+c = 2s \quad \text{—}(1)$$

Let Δ and Δ' be the areas of the old and new triangles respectively. Then

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)} \quad \text{(old area)}$$

$$\Delta' = \sqrt{s'(s'-2a)(s'-2b)(s'-2c)}$$

Now from (1)

$$s' = 2s$$

$$\therefore \Delta' = \sqrt{2s(2s-2a)(2s-2b)(2s-2c)}$$

$$\Delta' = 4\sqrt{s(s-a)(s-b)(s-c)}$$

$$\boxed{\Delta' = 4\Delta} \quad \text{—}(2)$$

∴ increase in the area of triangle

$= \Delta' - \Delta = 4\Delta - \Delta = 3\Delta$

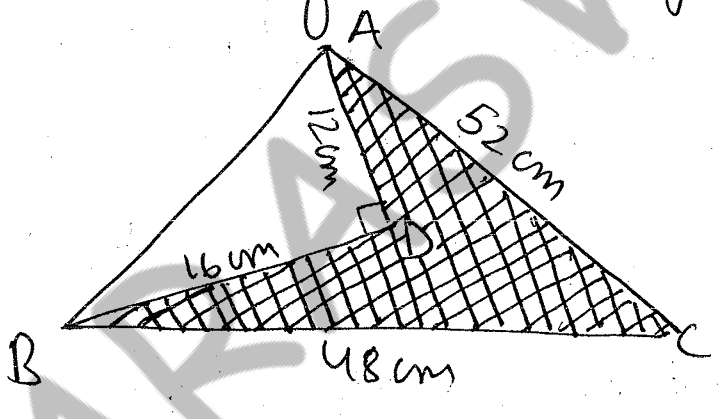
Hence, percentage increase in area

$= \dfrac{\text{Increase}}{\text{original}} \times 100$

$= \dfrac{3\Delta}{\Delta} \times 100 = 300\%$

Q6.) Find the area of shaded region

(Ans → $384 cm^2$)



Q7.) The length of sides of triangle are in ratio 3:4:5 and its perimeter is 144cm. Find the area of triangle and height corresponding to longest side. (Ans → $864 cm^2$, 28.8cm)

Q8) The perimeter of an isosceles triangle is 42 cm and its base is $\left(\frac{3}{2}\right)$ times each of the equal sides. Find length of each side of triangle, area of triangle and the height of triangle. (Ans → $71.42 cm^2$ & height = 7.94 cm)

⇒ Let equal side be x cm each

$\therefore$ base = $\frac{3}{2}$ × equal side = $\frac{3}{2}x$ cm

ATQ

Perimeter = 42 cm

$$\Rightarrow \underbrace{x + x}_{\text{two equal sides}} + \underbrace{\frac{3}{2}x}_{\text{Base}} = 42$$

$$\Rightarrow \frac{4x+3x}{2} = 42 \Rightarrow \frac{7x}{2} = 42$$

$$\Rightarrow x = \frac{\cancel{42}^{6} \times 2}{\cancel{7}}$$

$$\Rightarrow \boxed{x = 12 cm}$$

$\therefore$ Let $a = x = 12$ cm

$b = x = 12$ cm

$c = \frac{3}{2}x = \frac{3}{2} \times 12 = 18$ cm

(Now find area using HERON'S & then find height as done in Q3)

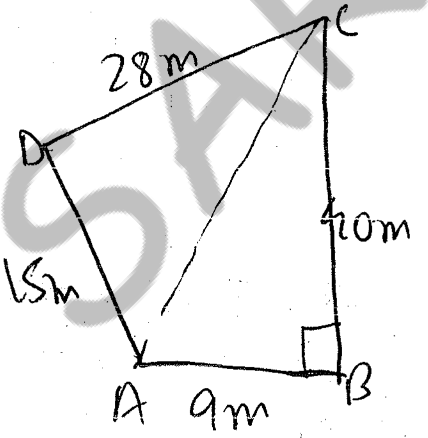
Q9.) In a ΔABC, $AB = 15cm$, $BC = 13cm$, $AC = 14cm$. Find area of ΔABC and hence altitude on AC.? (Ans → $84cm^2$, 12cm.)

Q10.) Find the area of Quadrilateral ABCD whose sides are 9m, 40m, 28m and 15m respectively and Angle between first two sides is a right angle. ($306m^2$ Ans.)

NOTE

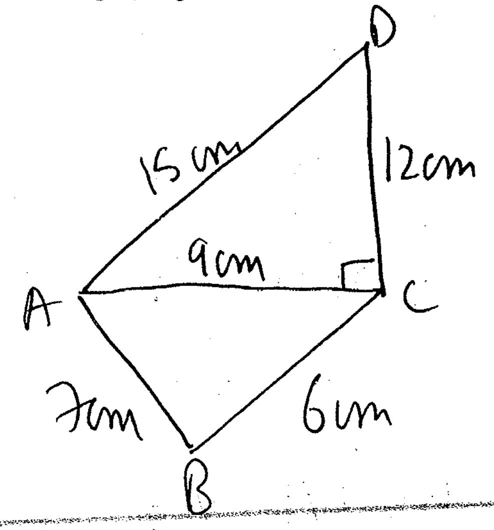
AREA of any Quadrilateral = sum of areas of two triangles.

=)



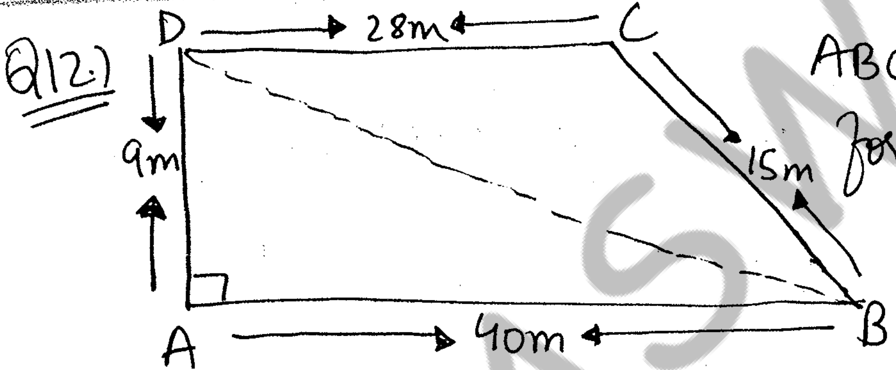
find area of ΔABC then area of ΔADC and add to find area of Quadrilateral ABCD

Q11.) Find the area of Quadrilateral ABCD, in which AB = 7cm, BC = 6cm, CD = 12cm, DA = 15cm and AC = 9 cm? (Ans → 74.98 cm^2)



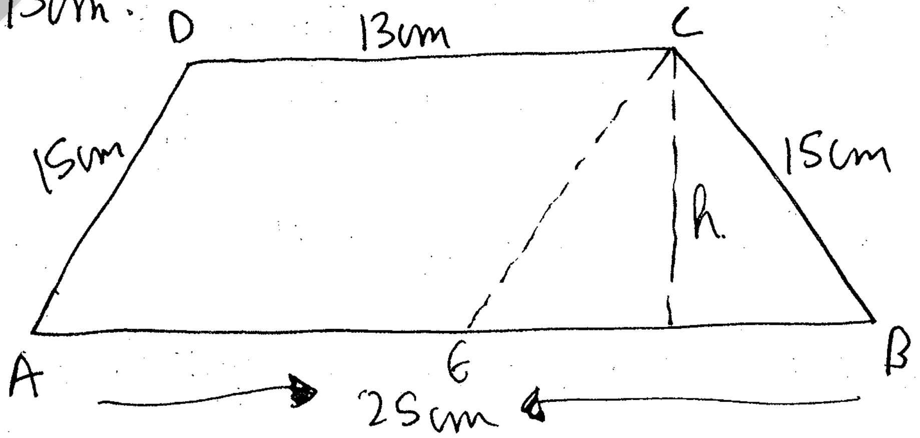
(Area of $\triangle ADC$ + area of $\triangle ABC$)

Q12.) ABCD is a field in the form of Quadrilateral whose sides are indicated in the figure. If $\angle DAB = 90°$, find area of the field?



(Ans → 306 m^2)

Q13) Find the area of trapezium whose parallel sides 25cm, 13cm and other sides are 15cm and 15cm.



Ans.) Let ABCD be the given trapezium in which AB = 25 cm, CD = 13 cm, BC = 15 cm and AD = 15 cm

Now CE ∥ AD

Now ADCE is a parallelogram in which AD ∥ CE and AE ∥ CD

AE = DC = 13 cm and BE = AB - AE

$\Rightarrow$ BE = 25 - 13 = 12 cm

In ΔBCE, we have

$$S = \frac{15+15+12}{2} = 21.$$

$$\text{Area of } \Delta BCE = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\Rightarrow \text{Area of } \Delta BCE = \sqrt{21(21-15)(21-15)(21-12)}$$

$$\Rightarrow \text{Area of } \Delta BCE = \sqrt{21 \times 6 \times 6 \times 9}$$

$$= 18\sqrt{21} \text{ cm}^2 \quad \text{— (1)}$$

Let h be the height of ΔBCE, then

$$\text{Area of } \Delta BCE = \frac{1}{2} \times \text{Base} \times \text{height} = \frac{1}{2} \times 12 \times h \quad \text{— (2)}$$

from ① & ②

$$\frac{1}{2} \times 12 \times h = 18\sqrt{21}$$

$$\Rightarrow h = \frac{18\sqrt{21}}{6} = 3\sqrt{21} \text{ cm}$$

Clearly, the height of trapezium ABCD is same as that of ΔBCE

Area of trapezium $= \frac{1}{2} \times (AB + CD) \times h$

$$= \frac{1}{2} \times (25 + 13) \times 3\sqrt{21}$$

$$= 57\sqrt{21} \text{ cm}^2$$

[Or find area of ||gm AECD $= b \times h$

$= 15 \times 3\sqrt{21}$]

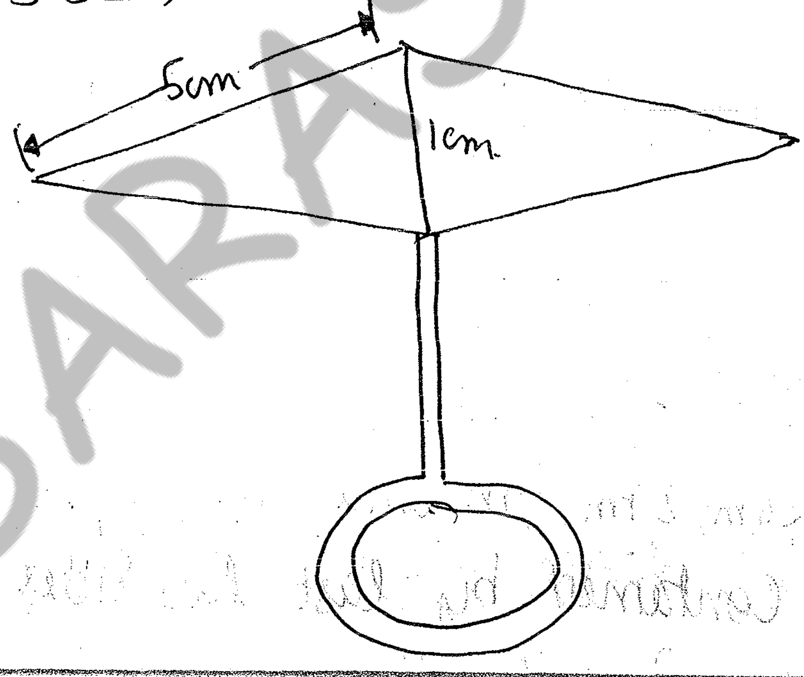
Q14.) The sides of a quadrangular field, taken in order are 26m, 27m, 7m and 24m respectively. The angle contained by last two sides is a right angle. Find its area

(Ans-) 375.8m^2)

Q15) A rhombus sheet, whose perimeter is 32m and whose one diagonal is 10m long, is painted on both sides at rate of ₹5 per m^2. Find cost of painting. (Ans. ₹625.00)

Q16.) Find area of rhombus whose perimeter is 80m and one of whose diagonals is 24m (Ans $384m^2$)

Q17) Find area of blades of magnetic compass ($\sqrt{11} = 3.32$) (Ans. $4.98cm^2$)



Q18.) Adjacent sides of parallelogram ABCD measure 34cm, 20cm and diagonal AC measures 42cm. Find area of parallelogram ($672cm^2$)

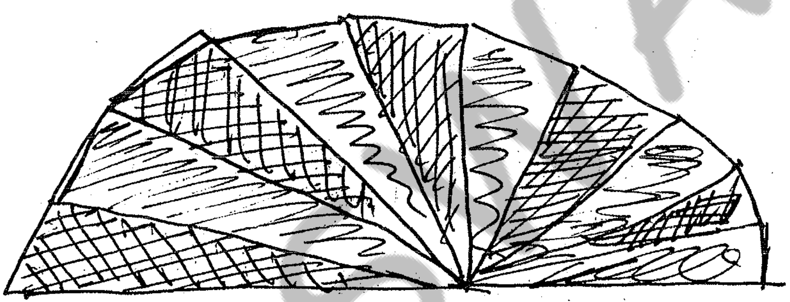
Q19.) A triangle and a parallelogram have the same base and same area. if the sides of the triangle are 13cm, 14cm and 15cm and parallelogram stands on base 14cm, find the height of parallelogram (Ans -> 6cm)

Q20.) Find area of Quadrilateral ABCD in which AD = 24cm, ∠BAD = 90° and BCD forms an equilateral triangle whose each side is equal to 26cm ($\sqrt{3} = 1.73$) (Ans -> 412.37cm²)

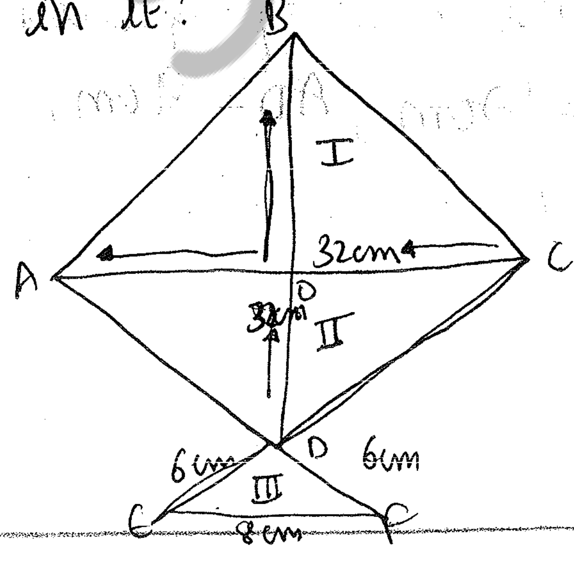
Q21) Two parallel sides of trapezium are 60cm and 77 cm and other sides are 25cm and 26cm. Find area of trapezium. (1644cm²)

Q22) Find the perimeter and area of Quadrilateral ABCD in which AB = 17cm, AD = 9cm, CD = 12cm, ∠ACB = 90° and AC = 15cm. (Ans 46cm, 114cm²)

Q23) A hand fan is made by stitching 10 equal size triangular strips of two diff. types of paper. The dimensions of equal strips are 25cm, 25cm and 14cm. Find area of each type of paper needed to make hand fan. (Ans $840 cm^2$ paper of each type)



Q24.) A kite in the shape of square with a diagonal 32cm and an isosceles triangle of base 8cm and sides 6cm each is to be made of 3 diff. shades how much paper of each shade has been used in it?



Since diagonals of square bisect each other at right angle.

$\therefore$ OA = OB = OC = OD = 16cm

Now,

Area of region I $= 2 \times$ area of ΔAOB

$\Rightarrow$ Area of region I $= 2 \times \frac{1}{2} \times OA \times OB$

$\Rightarrow$ Area of region I $= 2 \times \frac{1}{2} \times 16 \times 16 = 256\,cm^2$

Similarly,

Area of region II $= 256\,cm^2$

in ΔDEF, we have

$a = DE = 6\,cm$, $b = EF = 8\,cm$ and $c = DF = 6\,cm$

Let $2s$ be the perimeter of ΔDEF. Then,

$2s = 8 + 6 + 6 = 20 \Rightarrow \boxed{s = 10}$

$\therefore$ Area of region III $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{10 \times (10-6)(10-8)(10-6)}\,cm^2$

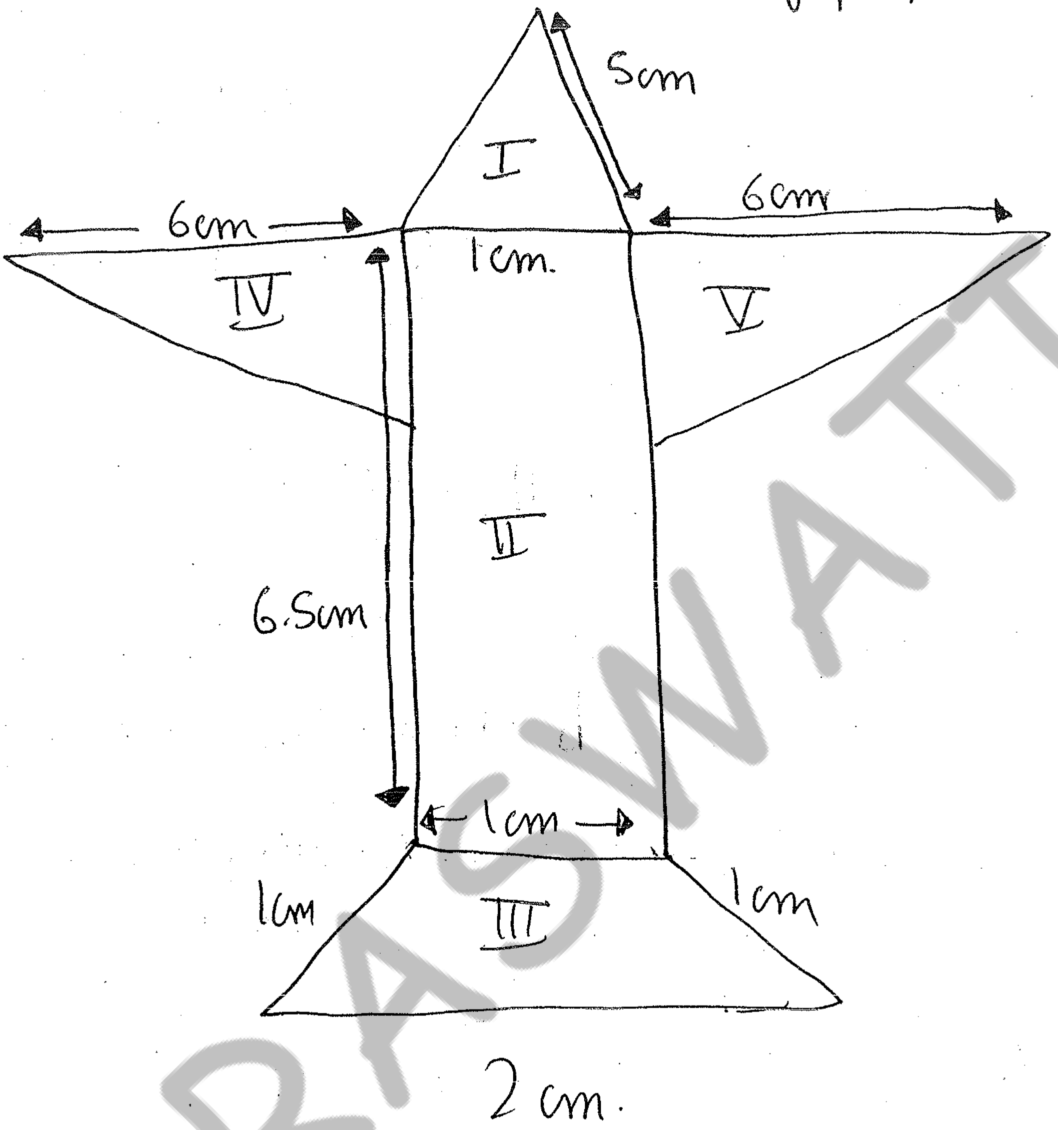
$= \sqrt{10 \times 4 \times 2 \times 4}\,cm^2$

$= \sqrt{5 \times 2 \times 4 \times 2 \times 4}\,cm^2$

$= 4 \times 2\sqrt{5}\,cm^2 = 8\sqrt{5}\,cm^2$

$= 8 \times 2.23\,cm^2 = 17.84\,cm^2$

Q25.) Radha made a picture of aeroplane with coloured paper as shown. Find total area of paper used



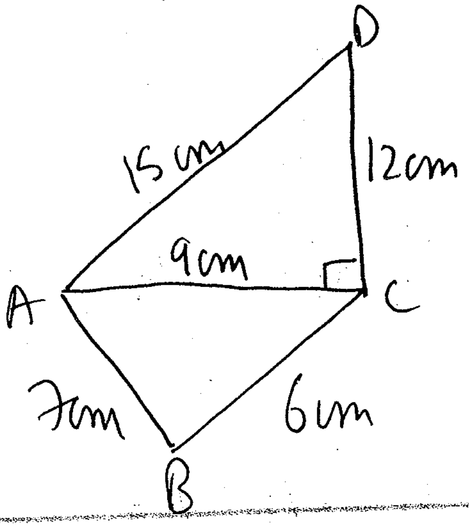
Area of Region I

$\Rightarrow$ Region I is enclosed by a triangle of sides $a = 5cm$, $b = 5cm$ and $c = 1cm$.

Let s be the semi-perimeter

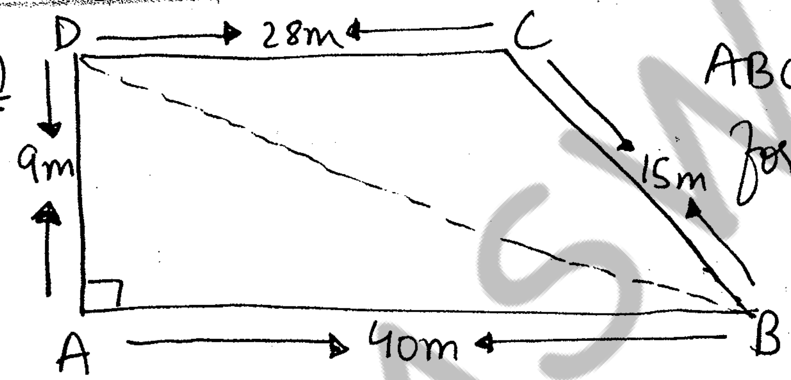
$$\therefore s = \frac{a+b+c}{2} = \frac{5+5+1}{2} = \frac{11}{2} \text{ cm}$$

Q11.) Find the area of Quadrilateral ABCD, in which AB = 7cm, BC = 6cm, CD = 12cm, DA = 15cm and AC = 9 cm? (Ans → 74.98 cm^2)



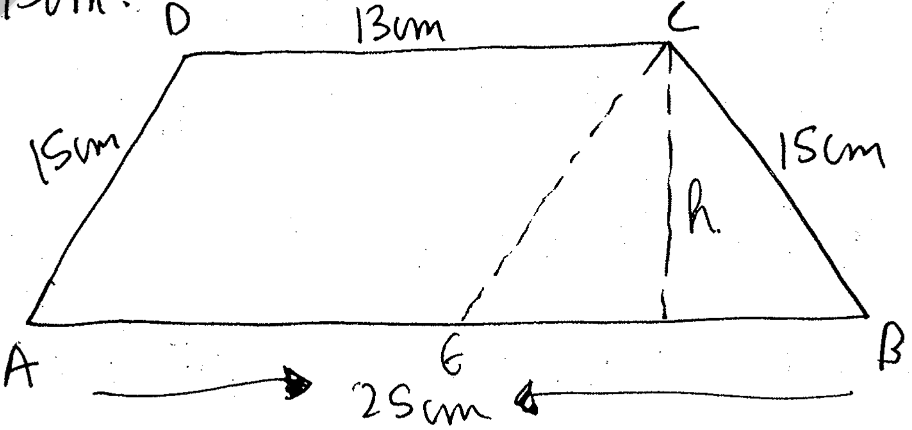
(Area of Δ ADC + area of ΔABC)

Q12.)



ABCD is a field in the form of Quadrilateral whose sides are indicated in the figure. If ∠DAB = 90°, find area of the field?
(Ans → 306 m^2)

Q13) Find the area of trapezium whose parallel sides 25cm, 13cm and other sides are 15cm and 15cm.



Ans.) Let ABCD be the given trapezium in which AB = 25cm, CD = 13cm, BC = 15cm and AD = 15cm

Now CE ∥ AD

Now ADCE is a parallelogram in which AD ∥ CE and AE ∥ CD

$AE = DC = 13cm$ and $BE = AB - AE$

$\Rightarrow BE = 25 - 13 = 12cm$

In ΔBCE, we have

$$S = \frac{15+15+12}{2} = 21.$$

$$\text{Area of } \Delta BCE = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\Rightarrow \text{Area of } \Delta BCE = \sqrt{21(21-15)(21-15)(21-12)}$$

$$\Rightarrow \text{Area of } \Delta BCE = \sqrt{21 \times 6 \times 6 \times 9}$$

$$= 18\sqrt{21}\ cm^2 \quad -(1)$$

Let h be the height of ΔBCE, then

$$\text{Area of } \Delta BCE = \frac{1}{2} \times \text{Base} \times \text{height} = \frac{1}{2} \times 12 \times h \quad -(2)$$

from ① & ②

$\frac{1}{2} \times 12 \times h = 18\sqrt{21}$

$\Rightarrow h = \frac{18\sqrt{21}}{6} = 3\sqrt{21}$ cm

Clearly, the height of trapezium ABCD is same as that of ΔBCE

Area of trapezium $= \frac{1}{2} \times (AB + CD) \times h$

$= \frac{1}{2} \times (25 + 13) \times 3\sqrt{21}$

$= 57\sqrt{21}$ cm^2

[Or find area of ||gm AECD $= b \times h$

$= 15 \times 3\sqrt{21}$]

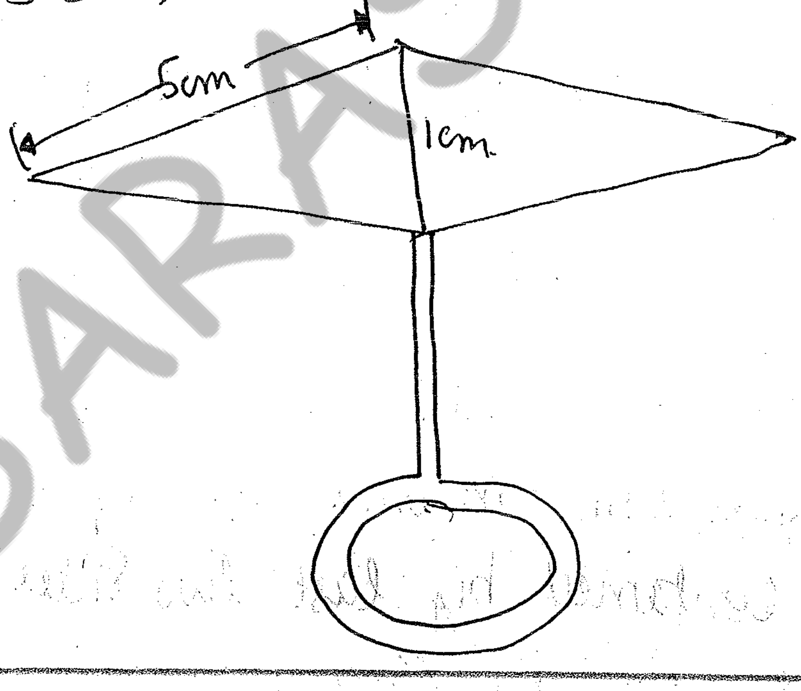
Q14.) The sides of a Quadrangular field, taken in order are 26m, 27m, 7m and 24m respectively. The angle contained by last two sides is a right angle. Find its Area

(Ans.) $375.8\,m^2$)

Q15) A rhombus sheet, whose perimeter is 32m and whose one diagonal is 10m long, is painted on both sides at rate of ₹5 per m^2. Find cost of painting. (Ans. ₹625.00)

Q16.) Find area of rhombus whose perimeter is 80m and one of whose diagonals is 24m (Ans 384m^2)

Q17) Find area of blades of magnetic compass ($\sqrt{11} = 3.32$) (Ans. 4.98cm^2)



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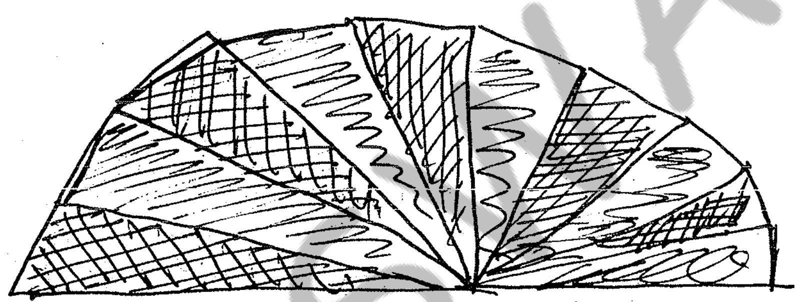

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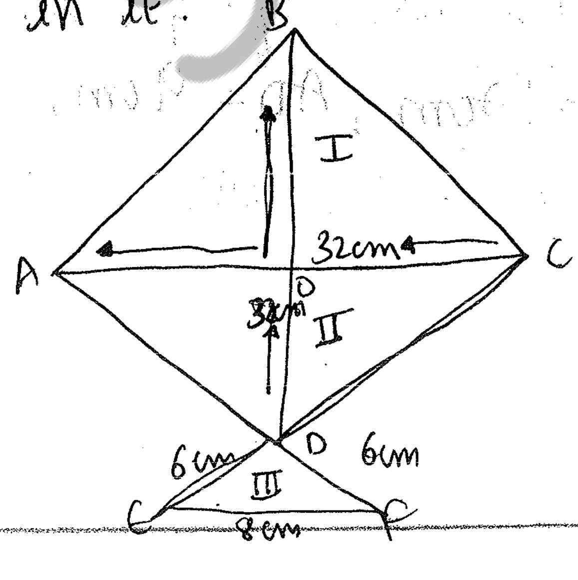

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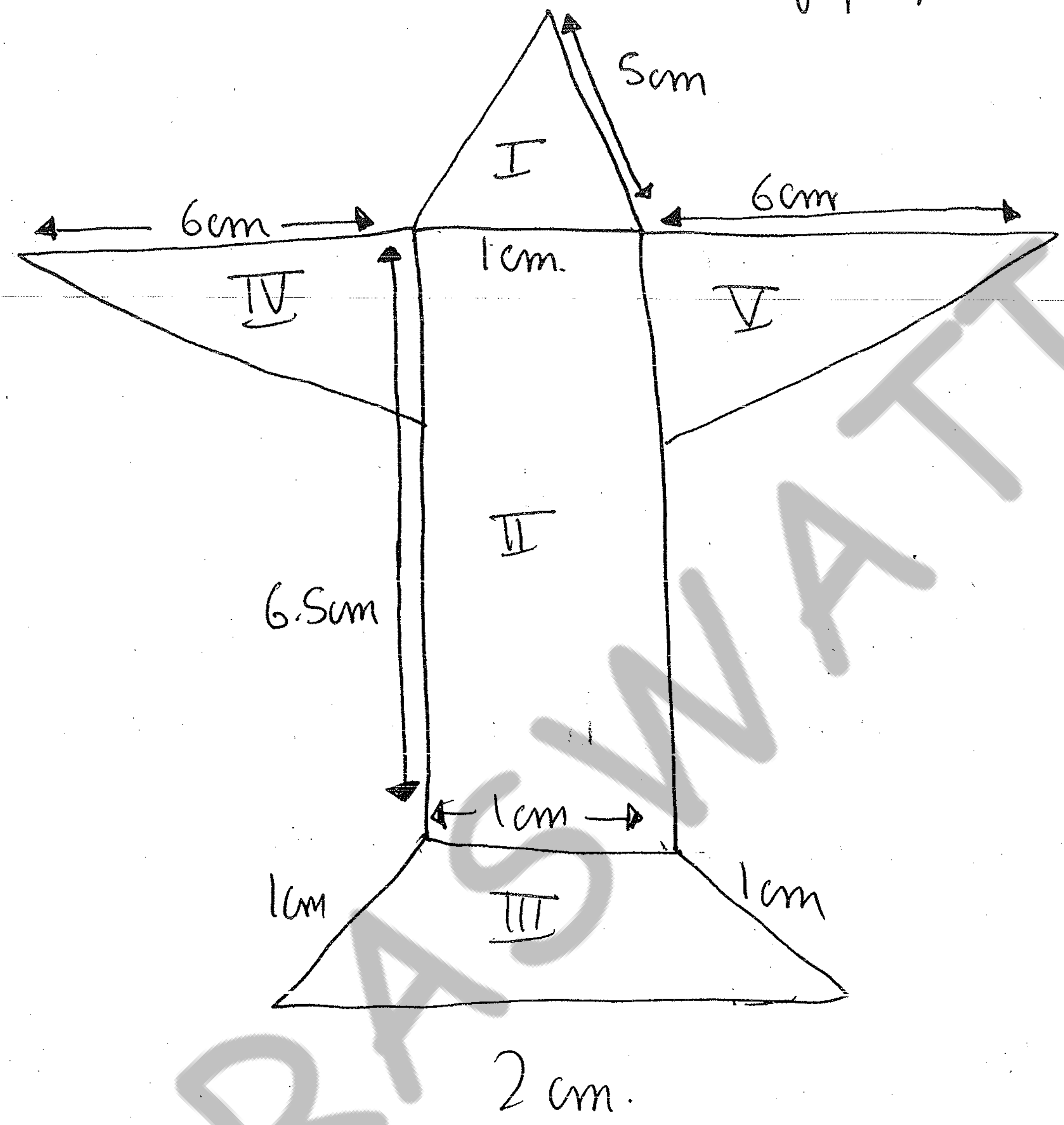

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$$\therefore s = \frac{a+b+c}{2} = \frac{5+5+1}{2} = \frac{11}{2} \text{ cm}$$

$\therefore$ Area of region I $= \sqrt{s(s-a)(s-b)(s-c)}$ cm^2

$$= \sqrt{\frac{11}{2} \times \left(\frac{11}{2}-5\right) \times \left(\frac{11}{2}-5\right) \times \left(\frac{11}{2}-1\right)} \text{ cm}^2$$

$\therefore$ Area of region I $= \sqrt{\frac{11}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{9}{2}}$ cm^2

$$= \frac{3}{4}\sqrt{11} \text{ cm}^2$$

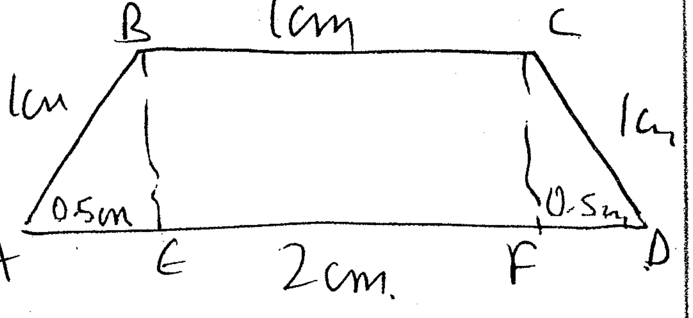
$$= \frac{3}{4} \times 3.32 \text{ cm}^2 = 2.49 \text{ cm}^2$$

Area of region II

Region II is a rectangle of length 6.5 cm and breadth 1 cm.

$\therefore$ Area $= 6.5 \times 1$ cm^2 $= 6.5$ cm^2

Area of region III



In $\triangle ABE$, we have

$$AB^2 = AE^2 + BE^2$$

$$1 = 0.25 + BE^2$$

$$\Rightarrow BE = \sqrt{0.75} = \sqrt{\frac{3}{4}}$$

Area of region III $= \frac{1}{2} \times (AD+BC) \times BE$

$$= \frac{1}{2} \times (2+1) \times \sqrt{\frac{3}{4}} \text{ cm}^2$$

$$= \frac{3\sqrt{3}}{4} \text{ cm}^2 = 1.3 \text{ cm}^2$$

Area of region IV

region IV forms a right triangle whose two sides are of lengths 6 cm and 1.5 cm

$\therefore$ Area of Region IV $= \frac{1}{2} \times 6 \times 1.5$

$$= 4.5 \text{ cm}^2$$

Area of region V

region IV and V are same

$\therefore$ Area of region V $= 4.5 \text{ cm}^2$

total area of paper used $= (2.49 + 6.5 + 1.3 + 4.5 + 4.5)$

$$= 19.29 \text{ cm}^2$$